Volume XXXV. *July, 1912.* *Number 1*

THE

PHYSICAL REVIEW.

ON THE VIBRATION OF A LECHER SYSTEM USING A LECHER OSCILLATOR, II.[1]

By F. C. Blake and Charles Sheard.

IN our former paper[2] we described some experiments performed on a Lecher system using a Paalzow-Rubens bolometer. The present paper is a continuation of that work but instead of a bolometer a thermal couple was employed. This has the advantage over the bolometer of greater constancy, because it is not dependent upon any auxiliary battery circuit. Its introduction led to some rather interesting results as will be seen further along in this paper.

Apparatus.

The apparatus, Fig. 1, was so mounted on a system of ways that the oscillator and Lecher circuits could each be bodily moved along the ways parallel to its own length. The end-capacity plates, eight in all, were made of tin 5 cm. in diameter and 0.3 mm. thick. The total length of the main receiver was roughly that of the bolometer circuit previously used. This system of ways afforded a ready means of changing the coupling at either end of the Lecher circuit. The precautions taken in the first paper for an uncontaminated wave-system were continued here, of course. Indeed, the two precautions used by Blake and Ruppersberg[3] of preventing stray radiation effects, though not used in our earlier paper, were again employed in the present experiments. The thermal couple was enclosed in a large metal box $2 \times 1\frac{1}{2} \times 1\frac{1}{2}$ ft., the lead wires being surrounded with 10-inch furnace piping up to within a foot of the end-capacity plates. The wires leading to the galvanometer were encased in an iron pipe, the pipe end near the galvanometer being stuffed

[1] We gratefully acknowledge our thanks to the trustees of the Elizabeth Thompson Science Fund for partial financial assistance in carrying on the work here reported.

[2] Phys. Rev., Vol. XXXII., page 533, 1911.

[3] Phys. Rev., Vol. XXXII., page 449, 1911.

with tinfoil and the pipe earthed. In this way the minimum readings were cut down at times to a value as low as 0.3 per cent. of the maximum for the free fundamental. While these precautions can affect the vibrations as to *type* not at all, nevertheless they are essential for accurate work.

As the work progressed it became evident that the degree of coupling at each end of the Lecher circuit and the relative lengths of the various circuits played such an important part in the character of the vibrations that it became tedious and unnecessary to continue the check receiver readings in order to obtain satisfactory results. All of the curves here shown except those of Figs. 8 and 13 were taken with the use of a main receiver alone. They cannot, of course, be as accurate as some of the curves of our first paper (*e. g.*, those of Fig. 9). Nevertheless we feel sure the reader will agree that they cannot be essentially in error on that account.

Resonance between the Oscillator and Lecher Circuits.

It was stated in the other paper that experiments were in progress where by employing a very long oscillator we hoped to free the Lecher system from the oscillator vibrations. This hope, however, was never realized except as it was realized through resonance conditions. We have worked with oscillators varying in length (expressed as the length AB, Fig. 1, with the wire stretched out straight) from 50 to 400 cm. and have

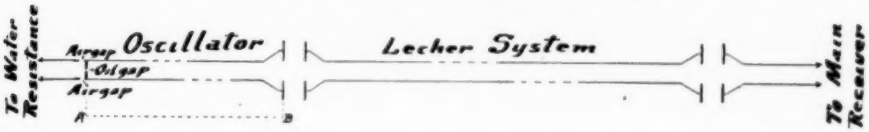

Fig. 1.

not been able to free the Lecher system from oscillator influence. Indeed it will be seen that we not only could not get rid of the oscillator influence but by substituting a thermal couple for the bolometer the receiver itself played a part in our curves. To what extent we have been able to differentiate the various effects one from another will show as the paper proceeds. Not anticipating any trouble due to the influence of the receiver upon the vibrations we started out by trying both very short and very long oscillators, but in each case the resultant set of vibrations was too complicated to be easy of explanation. We had proven that so long as the bridge length was inappreciable compared to the length of the Lecher wires, the vibrations are essentially longitudinal. Moreover, under such conditions we had shown that a circuit consisting of two

parallel wires closed at one end had possible frequency-ratios of 1, 3, 5, 7, etc., while for such wires open at both ends the possible ratios are 1, 2, 3, 4, etc. If, then, the oscillator circuit as to geometrical configuration were exactly equal to half the Lecher circuit it was figured that for a loose coupling resonance between the two circuits should obtain. When this was tried, however, it became apparent (Curve I., Fig. 2) that while we were near resonance, the oscillator was apparently too short. The data for the various circuits for all the curves here shown are collected in Table I. The oscillator 1st[1] occurs at 124.7, the free first at 119.4,

TABLE I.

Figure.	Curve Number.	Length of Oscillator.	Length of Lecher System.	Length of Receiver.	Coupling at Osc. End.	Coupling at Rec. End.
2	I.	120.0	240.0	313.5	4	4
	II.	125.5	240.0	313.5	6	6
	III.	131.2	240.0	313.5	6	6
5	I.	70.0	140.0	313.5	6	6
	II.	75.7	140.0	313.5	6	6
	III.	79.5	140.0	313.5	6	6
7	I.	90.0	169.6	313.5	6	6
	II.	92.5	169.6	313.5	6	6
8	I.	95.4	191.0	313.5	6	6
	II.	101.3	191.0	313.5	6	6
	III.	92.6	191.0	313.5	6	6
6	I.	141.0	169.6	313.5	6	6
	II.	141.0	169.6	313.5	3	3
	III.	141.0	169.6	325.5	6	6
	IV.	141.0	169.6	338.5	6	6
	V.	249.0	192.0	313.5	3	3
9	I.	97.0	186.0	313.5	6	6
	II.	102.0	186.0	313.5	6	6
	III.	97.0	186.0	313.5 (non-selective)	6	6

giving 5.3 cm. difference. Accordingly we added 5.5 cm. to the oscillator length and obtained Curve II., Fig. 2. When we added 5.7 cm. more, we obtained Curve III. The great superiority as to type of Curve II. over Curves I. and III. will be readily admitted. In Curve II. the oscillator and Lecher circuits are exactly in resonance and every peak

[1] While trying to unravel curves similar to those of Fig. 2 we became wholly converted to the notation already used by some, of calling the harmonics of a circuit by their frequency-ratios. Accordingly in this paper, the word "fundamental" is the same as the first vibration. The first overtone of the oscillator is called "oscillator 3d," the second overtone "oscillator 5th," and so on; the first overtone of the Lecher circuit is called "free 2d," the second overtone, "free 3d," and so on. This conduces wonderfully to clearness and we strongly recommend its universal adoption. To put our previous work (l. c.) under this notation, add 1 to the figures at the top of Fig. 1*a*, page 535. Similarly, for Blake and Ruppersberg's paper add 1 to the figures at the top of Fig. 4*a*, page 459.

except one can be accounted for as if it were a free vibration. The exceptional peak occurs at 91.5 cm. That it is due to the receiver there can be no doubt. We have named all the peaks for all three curves according to what light we have, and we believe that they are essentially correct. An interrogation point occurs after any that we deem doubtful. In Curve III. the crosses represent duplicate readings, taken when the oscillator was well deteriorated, hence the points are all low. They helped, however, to locate doubtful maxima—it should be borne in mind that the check receiver was not used and hence there was no way of differentiating a weak maximum from mere oscillator variation. For

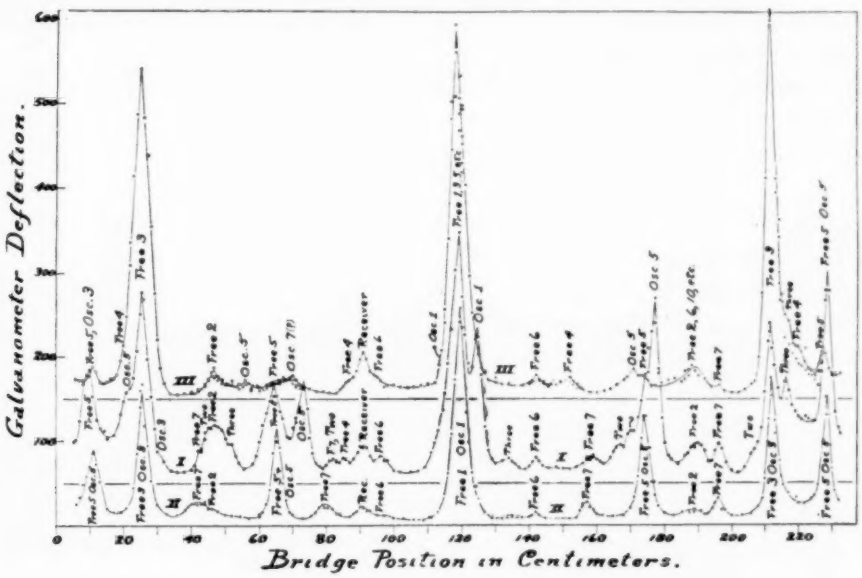

Fig. 2.

the longest oscillator (Curve III.) the oscillator 1st does not show as well as for the shortest oscillator (Curve I.). Rather it seems to swing in with the free first, making the maximum at 118 instead of 118.7. In Curve I. we see that all the possible oscillator and free vibrations are present to a greater or less degree leaving several maxima still unaccounted for. Fixing our attention upon the receiver maximum at 91.5 it is seen that other maxima, in all probability related to it, occur at 9.0, 50.5, 133.5, 175, 216.5, giving an internodal space of 41.5 cm. If we fix our attention upon the oscillator maximum at 124.7 we find other maxima apparently related to it at 41.7, 83.2, 166.2, 207.7, giving the same internodal space. Now, assuming the Lecher wires stretched out straight, the capacity-plates at their ends would read -1.3 and 238.7 cm., the

total wire length being 240 cm. Now, the free thirds occur at 25.7, 118.7, 211.7 with a mean internodal space of 93.0 cm. Adding half of this to 211.7 gives 258.2; subtracting 238.7 gives 19.5 cm. as the value of the end-capacity. Subtracting 41.5 from 41.7 gives 0.2; subtracting one half of 41.7 from 0.2 gives − 20.6, the oscillator end of the Lecher wires including end-capacity. Subtracting 1.3 from 20.6 gives 19.3 as the end-capacity. Adding 41.5 to 216.5 gives 258.0, in practical agreement with 258.2 above. Subtracting 238.7 from 258 gives 19.3 as end-capacity, checking 19.5 sufficiently well.

We are not at all clear as to the mode of vibration that causes the receiver peak to appear at 91.5 cm. Now, whenever we changed the Lecher wire length the receiver wires in general were not disturbed in their length nor space-position, nor were the plates of the Lecher system at the receiver end disturbed. The zero of the meter stick, however, was always put even with the oscillator plates of the Lecher system. It was observed, then (see many of the curves in this paper), that the receiver peak *always* came at 91.5 no matter what the length of the Lecher wires, nor that of the oscillator. The fairest explanation of this fact would seem to be as follows: Adding 19.3, 1.3 and 91.5 gives 112.1 cm. This seems to be the quarter-wave-length, then, of the receiver. It will be shown later that when the receiver length is increased x cm. the receiver-peak on the Lecher wires is displaced $4/13x$ cm. toward the receiver end of the Lecher wires. Now, the total receiver length was 627 cm. roughly from plate through the thermal couple to plate. The wires from the plate to the thermal junction were practically parallel and at a distance apart of 30 cm. through the greater part of their length. The Lecher wires on the other hand were only 5 cm. apart. The receiver wires would accordingly have less capacity per unit length than the Lecher wires were it not for the fact that they were surrounded with the 10-inch furnace-piping. The extra capacity due to the piping slightly more than overcomes the decrease in capacity due to their greater distance apart. Now, $3 \times 112.1 = 336.3$ cm. while half of 627 $(= 313.5) +$ $19.3 = 332.8$ cm., the difference being due to the capacity differences just spoken of. If the fraction above, 4/13, is called 1/3, we can say that the peak at 91.5 is due to the first overtone of the receiver. This is borne out by the fact that when the Lecher wires were cut to a length so as to be in unison with the receiver wires, thus bringing this receiver peak and the free first together the tone was unusually strong; clearly the mode of vibration then must have been as in Fig. 3. Moreover, $41.5 \times 8 = 332$ cm., roughly the same as either of the numbers above. It seems then that the maxima at 9.0, 50.5, 92 (91.5), 133.5, 175, 216.5

(some of them are marked "Three") can be accounted for as a submultiple of the combined circuit formed from the three circuits taken together (*A*, Fig. 4), while the maxima at 41.7, 83.2, 124.7, 166.2, 207.7 (some of them are marked "Two") in the same way are due to the oscillator and Lecher circuits combined (*B*, Fig. 4). There is nothing in the nature of things against this explanation, unusual as it may seem. It

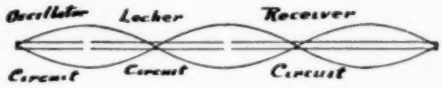

Fig. 3.

ought to be said that the greatest amount the figures have been "forced" in the one case is 0.5 cm., 92 instead of 91.5 cm., and 0.7 cm. in the other, 207.7 instead of 207.0 cm. This leaves a solitary maximum at 170 unaccounted for in Curve I., Fig. 2. It should be noted that the receiver peak at 216.5 is unusually strong, suggesting an internodal space of 3×41.5 or 124.5 cm. as shown in *A*, Fig. 4. Note also from *B*, Fig. 4, that the last node occurs 5.9 cm. behind the spark-gap. Now, this number is in good agreement with the amount that has to be added to the oscillator to make it in resonance with the Lecher system, viz., 5.5 cm.

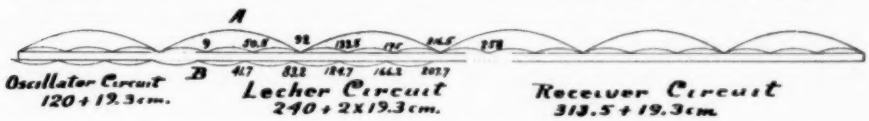

Fig. 4.

We believe that we have in Curve I., Fig. 2, an unusual opportunity to show that under certain conditions two of the circuits act together as a single circuit. Note first that the end-capacity, 19.3 cm., is roughly one half of the internodal space 41.5 cm. and that 3×41.5 is very close to 125.5 cm. This probably accounts for the fact that this internodal space is what it is. In other words, note that in *B*, Fig. 4, the potential loop spans the two capacity plates between the oscillator and Lecher circuits. (The equivalent wire length of these two plates, 38.6 cm., is drawn as a dotted line in the figure.)

Table II. shows clearly from all three curves of Fig. 2 how from the relative displacements between the various harmonics of the oscillator and Lecher systems one can arrive at the length of the oscillator that should respond to the Lecher system. The first set of figures in column 9 show a remarkable agreement, the mean being that actually used to obtain Curve II.

TABLE II.

Figure.	Curve Number.	Frequency Number n_0	Position. Osc.	Position. Free.	Difference δ.	$\delta \times \frac{n}{n'}$.	Actual Osc. Length.	Cal. Osc. Length for Resonance.	Mean.	Osc. Length Including End-capacity.	Lecher System Length Including Capacity.	Oscillator "Wave-length."	Ratio.
2	I.	1	124.7	119.4	5.3	5.3	120.0	125.3					
		5	21.0	11.0	10.0	5.6		125.6					
		5	73.0	65.0	8.0	5.7		125.7					
	II.						125.5	125.5					
	III.	5	57.0	65.0	8.0	5.7	131.2	125.5					
		5	170.0	173.5	3.5	5.8		125.4	125.5	144.8	278.6	579.2	2.08
5	I.	1	75.0	69.6	5.4	5.4	70.0	75.4					
		5	44.0	39.0	5.0	3.6		73.6					
		5	103.0	101.5	1.5	2.5		72.5					
	II.	3	10.0	12.0	2.0	1.2	75.7	74.5					
	III.	1	66.5	69.6	3.1	3.1	79.5	76.4					
		3	7.5	12.0	4.5	2.7		76.8	74.8	94.1	178.6	376.4	2.11
7	II.	5	40.5	45.0	4.5	3.2	92.5	89.3	89.3	108.6	208.2	434.4	2.08
8	II.	5	2.5	5.0	2.5	1.4	101.3	99.9	99.9	119.2	229.6	476.8	2.08
9	I. and III.						97.0	97.0					
	II.	5	43.5	50.0	6.5	4.6	102.0	97.4	97.2	116.5	224.6	466.0	2.08

n' is the number of quarter wave-lengths between any given maximum and the receiver end of the Lecher system including end-capacity.

Considering the general features of the curves of this figure one remarks that the relative strength of the free thirds increases as the oscillator length increases through the resonance point while the reverse is true for the free 5ths. Moreover, whereas, in Curves I. and II. the free 7ths are quite strong, with the free 4ths practically absent, in Curve III. we find the 7ths practically gone with the free 4ths coming in quite strong. One is tempted to inquire whether Curve II. will not submit readily to Abraham's simple theory, but it is plain upon inspection that the free 5ths are too strong and the free 1st is too weak compared to the free 3ds to make a comparison with theory worth while. We attribute this to the receiver influence. If this could be got rid of, one should expect agreement between theory and experiment for the *odd* harmonics in the case of resonance between the oscillator and Lecher circuits. That the oscillator plays an important part can be seen by comparing the relative intensities of the even with the odd harmonics in the curves of Fig. 2.

RATIO BETWEEN THE OSCILLATOR WAVE-LENGTH AND THE LECHER WIRE-LENGTH.

We have shown clearly from the curves of Fig. 2 and from Table II. that the oscillator circuit must be longer than the half Lecher circuit in order to be in resonance with it. The ratio between the oscillator "wave-length" and the Lecher wire-length from Fig. 2 is 2.08, obtained as follows: $125.5 + 19.3$ is 144.8 cm., $240 + 2 \times 19.3 = 278.6$ cm., $(4 \times 144.8)/278.6 = 2.08$. For both circuits we have neglected the bridge length, 5 cm., for we have assumed the currents strictly longitudinal. That this is a safe assumption will be partly conceded, we think, from the results already presented. Further justification for this assumption is presented later in this paper.

Having to our surprise found the oscillator length four per cent. greater than the Lecher half-length for resonance we proceeded to try out other oscillator and Lecher lengths. Three curves similar to those of Fig. 2 are shown in Fig. 5. Tables I. and II. give the data needed to understand the curves. That Curve II. is again a better resonance curve than either I. or III. will be readily conceded. However, the free third at 12 cm. shows itself mingled with something else at 10 cm.; in other words, the oscillator was too long by a little. When it was cut down to 73.5 cm. the resonance was exact. We mention a few of the more important things about these curves. In their general features they are entirely similar to those of Fig. 2. As the oscillator is lengthened certain harmonics decrease and others increase in intensity. However, in III., Fig. 5, the oscillator shows itself very markedly, whereas in III., Fig. 2, its

maxima were relatively weak. Inspecting the free third at 127 we see how when the oscillator is too short it is pulled over to 129, although when the oscillator is too long the peak still seems at 127. However, the free 5th at 101.5 is pulled up to 103 by a short oscillator and down to 99 by a long oscillator. This sort of displacement is surely in general to be

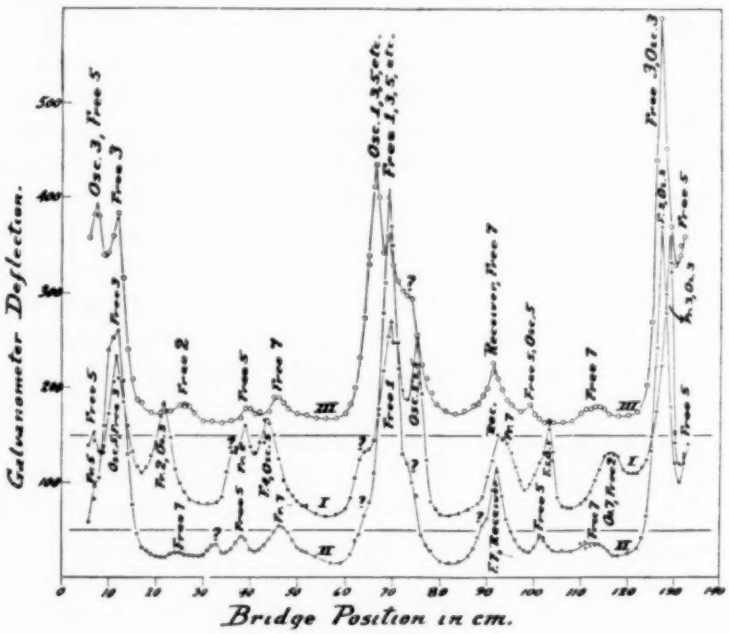

Fig. 5.

expected. We don't know that we can explain the maxima marked with an interrogation point. Apparently again some are due to the oscillator and some to the receiver. The receiver peak at 91.5 shows strongly in all three curves. It is worth remarking that peaks may be said to appear at 10.5, 37.5, 64.5, 91.5, 118.5 with an internodal space of 27 cm. Adding 27 cm. to 118.5 gives 145.5. Now for a 6 cm. coupling the end-cap is 18.6 cm. which when added to the end of the wire 138.7 gives 157.3 as the end of the wire including end capacity. Subtracting 145.5 from 157.3 gives 11.8 cm. Subtracting 11.8 from 336.3 gives 324.5 cm. Dividing this remainder by the internodal space of 27 cm. gives exactly 12. In other words the Lecher circuit and receiver circuit again behave as a single circuit. We suspect that the reason the oscillator peak at 66 is so strong in Curve III., Fig. 5, whereas in Curve III., Fig. 3, the same peak at 113 scarcely shows, is due to the presence of a receiver peak at 64.5, while in Fig. 2 the nearest receiver peak was 20 cm. away.

In Fig. 5 this leaves a peak at 33 in Curve II. and a stronger peak at 73 in both II. and III. unexplained. The fact that the latter remains stationary though the oscillator length varies points to its being some sort of a receiver peak. We will show presently that the ratio of oscillator "wave-length" to Lecher wire-length is in all cases investigated 2.08, but before we do this it is better to discuss the influence of the receiver upon our results.

INVESTIGATION OF THE RECEIVER INFLUENCE.

Having found that the receiver was exerting its own influence upon the wave system on the Lecher wires it became necessary to study its effect thereon. We first tried to see what effect a small change in length of the receiver would have. For a given receiver length the receiver peak had stood consistently at 91.5 cm. through a large number of variations of oscillator- and Lecher-lengths. The various changes we tried are recorded in Fig. 6. With an oscillator length of 141 cm., a Lecher length of 170 and a coupling of 6 cm. at each end we obtained Curve I., with the free first at 83.5 cm. and the receiver peak at 91.2 cm. When the coupling was made 3 cm. at each end the receiver peak predominated over the free first, both peaks being displaced toward the oscillator because of the closer coupling, as shown in Curve II. Adding 12 cm. to each lead wire of the receiver gave Curve III. for the receiver peak with the maximum at 95 cm., coupling 6 cm. again. Adding 25 cm. to each side of the receiver changed Curve I. to Curve IV. In each of these last two cases the receiver peak is displaced 4/13 of the amount added to each side or 2/13 of the total increase in receiver-length. Note that the displacement is *away* from the oscillator, and that the receiver peak gets less in intensity as the receiver length is increased. Curve V. is inserted for the following reason. Earlier in the work under different experimental conditions we had obtained Curve V. and had naturally

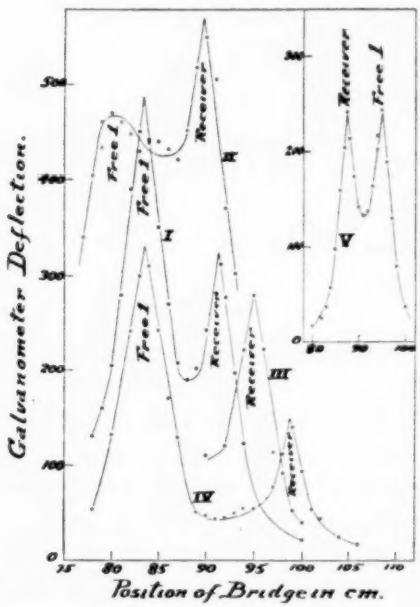

Fig. 6.

attributed it to the splitting up of the fundamental tone into two peaks because of close coupling (3 cm.). This was the more naturally done because the two peaks were identical in height. Later by work similar to the above we proved this was entirely wrong, one of the two peaks being due to the receiver, as marked. But it took us several weeks to run this down and to thus find out our error.

In Fig. 7, Curve I., while still investigating the receiver peak at 91 cm. we had the oscillator 90 cm. long, the Lecher wires 170 cm., coupling 6 cm. at both ends. Thinking the oscillator too short as revealed in the free third at 16 cm. we added 2.5 cm. to it, and also added 120 cm. to each side of the receiver thus getting Curve II., Fig. 7. But a study of

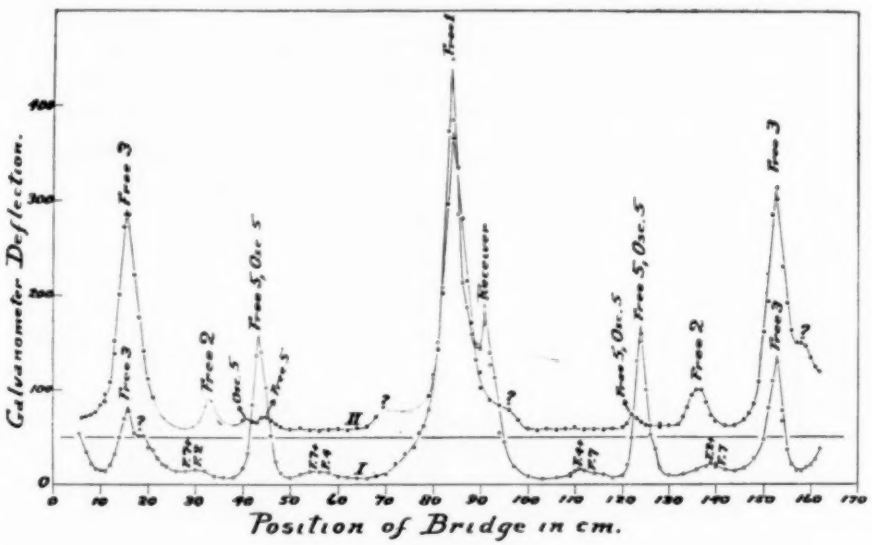

Fig. 7.

both curves (see Table II.) shows that the oscillator even in Curve I. was already too long, the maximum at 19 being due to some cause other than the oscillator 3d. A close study of the curves of this figure shows the displacing of the different maxima by the various influences able to operate. We have marked the maxima which unquestionably belong to the different harmonics even though displaced. Note that in Curve II. the receiver peak at 91 has disappeared because of the increased receiver length. From Fig. 6 this peak should have been displaced from 91 to 128 thus: 4/13 of 120 = 37. 91 + 37 = 128. But it apparently is not present. Either it has weakened so in intensity because of the great increase in length of the receiver or the ratio 4/13 does not hold for the displacement. Indeed for purposes of explanation of some of the maxima

as being due to all three circuits combined swinging as a single circuit we have already (rather arbitrarily) taken this ratio as 1/3 instead of 4/13.

Now, early in the work, using the check receiver, we had obtained Curves I. and II. of Fig. 8 and without the use of the check receiver, Curve III. Tables I. and II. show the necessary data. At the time they were taken the peak at 91 cm. was not understood. Curve I. was taken when the oscillator circuit as to geometrical configuration was exactly equal to half of the Lecher system. It does not show the double peak for the free and oscillator fifths at 50 cm. that Curves I. of Figs. 2 and 5 and Curve II. of Fig. 7 do under similar circumstances. Indeed except for the receiver peak at 91, Curve I. is such a satisfactory curve that it would seem that the ratio between the oscillator "wave-length" and the Lecher wire-length ought to be exactly 2.00 instead of 2.08. This leads us to ask the question, May not this ratio 2.08 be due after all to the receiver influence since, when the receiver is so nearly in tune with the free first as in the curves of Fig. 8, the free maxima are apparently the

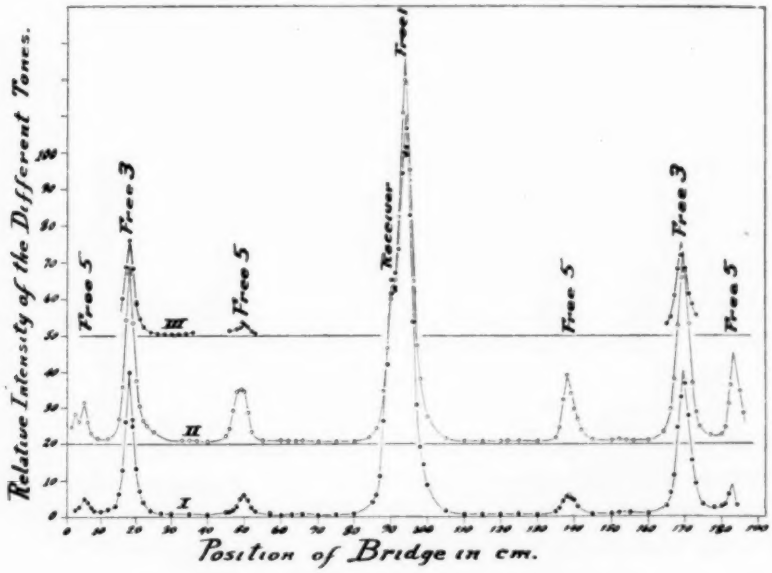

Fig. 8.

only ones present? Curve II. helps us to answer the question. Here the oscillator was longer and Table I. shows that the length for resonance is 99.9 cm. while the Lecher length was 191 cm. giving 2.08 for the ratio. This makes us believe that if a reading had been taken at 55 cm. in Curve I. it would have been at a maximum due to the oscillator fifth. Indeed the maximum at 138 cm. may easily be said to consist of two, one at

138.0, the other at 140 cm. Adding $5/3 \times 2$ to the oscillator length gives 98.7 making the ratio 2.06. The mean of these is 2.08 as before. Curve III. was partly taken when the oscillator was too short, 92.6 cm.

Resonance between All Three Circuits.

One has only to glance at the curves of Figs. 2, 5, 7 and 8 to convince himself that as the receiver peak falls in more closely with the free first the ratio of maximum to minimum rapidly increases. Accordingly we sought to have exact resonance between the Lecher and receiver circuits and obtained it in Curve I. (black circles (•)) of Fig. 9. The only

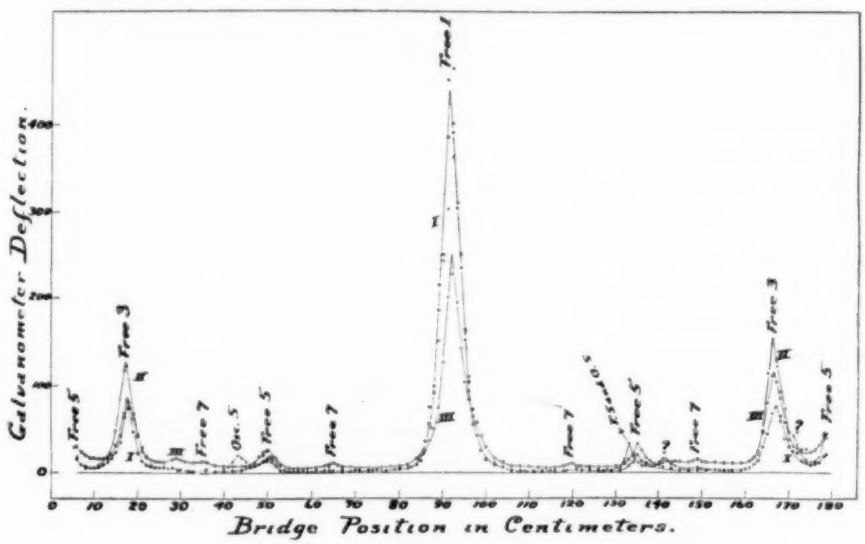

Fig. 9.

maxima present are the free first, third and fifth. But it is plain upon inspection that the free first is far too high in comparison to the other two harmonics for Abraham's theory to apply. Indeed we should not expect it to apply. For the first overtone of the receiver swings in with the free first to make it far too strong. We see no reason why when the receiver fundamental swings in with the free first its other tones should not resound to the corresponding free tones and thus render Abraham's theory applicable—on the assumption of course of proportionate damping for the different tones. We propose to try this soon.

If now the ratio 2.08 is really due to the oscillator and not to the receiver, then should the oscillator be lengthened somewhat the double peak, one due to the oscillator fifth, the other to the free fifth, should show in the 50 cm. region. Curve II. (crosses (×)) shows the double

peak. The oscillator fifth occurs at 43, the free fifth at 50. Now $7 \times 5/7 = 5$, the amount the oscillator was actually lengthened. This proves conclusively, we think, that the ratio 2.08[1] above is legitimate and in no sense due to the receiver influence; for the receiver, being in tune with the Lecher circuit, has no chance to act. Moreover we have obtained the ratio 2.08 under such a variety of conditions that the reader will scarcely be disposed to question it. It behooves us to ask ourselves then why should the oscillator have to be four per cent. longer than half the Lecher circuit length in order to cause it to respond at its best? We are aware that because of the spark-gap the damping in the oscillator circuit is greater than in the Lecher circuit, and that so far in our problem we have assumed the damping the same in the two circuits. Moreover, it is well known that the effective wave-length of an oscillation is increased by increased damping, and that accordingly the Lecher half circuit ought to be longer than the oscillator circuit to respond to it, whereas our experiments give exactly the reverse. Now it is rather remarkable that this ratio 2.08 between the oscillator and Lecher circuits is the same that both Blake and Sheard and Blake and Ruppersberg found for the ratio of the Lecher wave-length to the Lecher wire-length for a very loose coupling. If we divide the one ratio by the other we get the ratio between the oscillator "wave-length" and the Lecher system wave-length, viz., unity. This is the very thing we ought to expect. And yet to get the oscillator "wave-length" we have multiplied its own length by 4, whereas for the Lecher system we multiplied its half-length by 4.16. Why shouldn't the oscillator circuit show this "end-correction" of 4 per cent. as well as the Lecher circuit? This all points, it seems to us, to the conclusion that this end-correction is intimately bound up with the question of the relative distribution of the three circuits concerned. And yet we have satisfied ourselves by experimentation that loosening either coupling beyond 6 cm. has no effect upon this ratio of 2.08 between the oscillator and Lecher circuits. There *is* one consideration that hasn't yet been taken account of, viz., What effect upon the effective oscillator length does the inpour of energy from the induction coil have? Perhaps an entirely different method of experimentation would throw some light upon this knotty question. This work shows, furthermore, that for two parallel wires without end-capacities the ratio 2.08 obtained by us and by Blake and Ruppersberg is the limiting ratio for wave-length to wire-length of the Lecher system. This settles the question raised by us in our

[1] In Table II. for all the curves except II. and III. of Fig. 5 this ratio is 2.08. The mean oscillator length is shown in the table as 74.8. This is unquestionably too high for when the actual length was 73.5 cm. a sharp resonance was obtained for the free third at 11.5 cm. Using this length, the ratio comes out 2.08.

first paper, whether the Poincaré-Abraham ratio of 2.00 could ever be reached. We think now that it cannot, the reason being that there is an "end-correction" in the electrical case just as there is in the acoustical case, the amount of the correction being 4 per cent. Doubtless for wires of different diameters the correction varies somewhat, possibly shading off to zero for very fine wires which are *very* long compared to their distance apart.

Having found the presence of receiver influence to a greater or less degree in almost all of our curves, and having shown that the ratio of maximum to minimum is very much increased when the receiver is brought into exact tune with the other two circuits it became interesting to plot the ratio of maximum to minimum against the Lecher wire-length holding the receiver-length constant but keeping the other two circuits in tune with each other. The curve obtained is given in Fig. 10, the data for it being obtained from the other curves of this paper. As already remarked the maximum ratio was 325 for the first overtone of the receiver in tune with the other two circuits. It seems fair to expect that this ratio could be increased two or three fold by bringing the fundamental of the receiver into tune with these circuits. We expect to try this at the first opportunity.

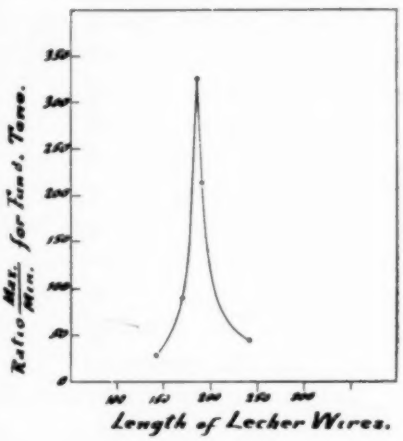

Fig. 10.

In spite of the large effect played by the receiver in our experiments as revealed especially in Fig. 10, the reader will not question, we believe, that the receiver does in no way affect the *type* of the wave system on the Lecher wires. On the other hand it surely *can* affect the relative intensity of the different harmonics, according as the receiver fundamental or some of its overtones falls in or out of phase with them. It became desirable accordingly to secure a resonance curve where the receiver might be said to be "non-selective." A first approximation to such a receiver was made by substituting for the short thermal junction wires two long ones, each 5.5 inches in length, care being taken, however, to leave the total length unchanged. That the new receiver was essentially "non-selective" was proven in the following way. 58 cm. was temporarily added to each receiver wire. The receiver peak if present ought to show itself at 110 cm. But there was apparently no trace of it, although readings

were taken by 2 cm. steps from 90 up to 130 cm. The resultant curve for this "non-selective" receiver is shown as Curve III., Fig. 9 (white circles (○)). Two things are evident by comparing Curves I. and III. of this figure. First, changing from a selective to a non-selective receiver does not affect the ratio 2.08 for the oscillator "wave-length" divided by the Lecher wire-length. Second, the free first is decreased to a normal value while the higher tones are slightly increased.

One is tempted, naturally, to try the simple theory on this Curve III. But a close inspection of this curve or indeed of any of the curves of this paper makes it clear that the agreement between theory and experiment is not good. Fig. 11 makes this plain. Calculating from the internodal

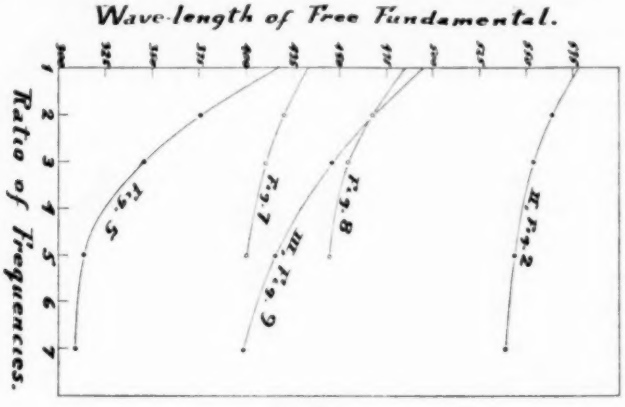

Fig. 11.

spaces for the different tones the wave-length of the free fundamental, using the resonance curves wherever possible as showing the least displacement of the various maxima, we obtain Fig. 11 and it is plain that in no case are the various tones strictly harmonic. We are in no position at present to state the reason for this. However, one or two remarks are in order, perhaps. Blake and Ruppersberg showed (see their Fig. 6) that too close a coupling caused an inharmonic relationship amongst the different frequencies, but unlike the present case the higher the tone the greater the fundamental wave-length. It could easily be that too loose a coupling would reverse this. At any rate our experiments, all taken with a very loose coupling, viz., 6 cm. at each end of the Lecher wires, demonstrate clearly enough that the higher the tone the lower the wave-length of the free fundamental. It is to be noted, however, that more than one cause is present, for the curves of Fig. 11 do not all slope equally fast, nor does the slope change uniformly with increasing Lecher wire-length.

Are the Currents in a Lecher System Longitudinal or Transverse?

In an important paper Lecher[1] showed twenty-one years ago that if a double bridge be placed on a system of parallel wires and the wires be then cut between the two parts of the bridge, the electric potential at the ends was very slightly diminished if at all. For his potential detector Lecher used a nitrogen discharge tube, which, of course, in the nature of things is not a satisfactory quantitative instrument.

It was a comparatively easy matter for us to repeat Lecher's experiment and so while trying to clarify our notions we did so early in the work. Our apparatus was accordingly that illustrated in Fig. 7 of the former paper except that the coupling was kept constant at − 2 cm. at each end of the Lecher wires throughout the work reported below. The bolometer was used as detector and a check receiver was employed.

First, we took readings both for a single bridge and for a double bridge placed in the middle of the Lecher system. The readings were the same, as we expected. Then, we soldered the double bridge to the Lecher wires and cut the Lecher wires between the parts of the double bridge. Again the readings were the same before and after cutting. We then replaced the Lecher wires by a new set consisting of two wires so arranged that the middle portion of each for a distance of 5 cm. constituted one half of the double bridge. In this way the Lecher system was divided into two equal parts which could be readily separated from each other. When the two parts of the double bridge were separated from each other by a piece of insulating tape 0.3 mm. thick, the potential at the bolometer end of the Lecher wires was 1 per cent. less than when touching. When a short piece of copper wire was soldered across each end of the double bridge parallel to the Lecher wire length, the potential was just the same as when the two parts of the bridge were touching. For these data a series of readings was taken using the check receiver. The same method was used for the curve of Fig. 12.

We then became interested to see how fast the potential at the bolometer end of the Lecher wires dropped off as the two parts of the system were separated. It was this work that forced us to resort to the metal protection of the bolometer and its lead wires. The results are shown in Fig. 12. In plotting the curves the radius of the wire, 0.04 cm., was taken into account, the abscissas representing the distance between the axes of the two wires of the double bridge. Curve I. was taken in the same manner as all the other curves of our first paper, viz., without the bolometer's being protected against stray radiation. When the bolometer

[1] Lecher, Wied. Ann., 41, p. 850, 1890.

was enclosed in a metal box and its lead wires in 10-inch furnace-piping we obtained Curve II. Plotting $\log_e$ (abscissas $\times$ 10,000) against the ordinates we get Curve II. *a*, showing that the potential drops off as the logarithm of the distance between the two parts of the bridge. For distances beyond 2 cm. the Curve II. *a* ceases to be a straight line. Now a considerable time after this Curve II. was taken we discovered that

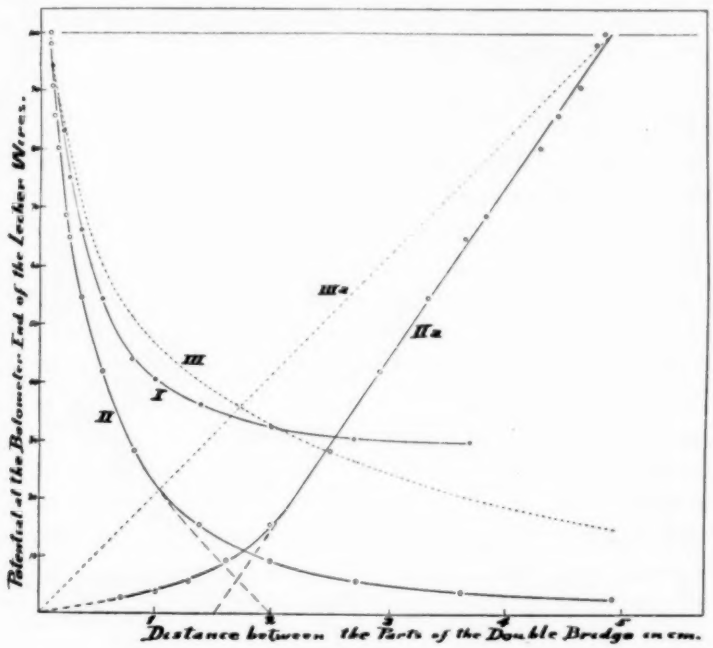

Fig. 12.

some stray radiation was striking the unprotected galvanometer lead wires and running back to upset the bolometer balance. This might have been sufficient to account for the bending of Curve II. *a* for the larger distances, and so we have continued the straight portion of II. *a* as a dotted line until it cuts the axis of abscissas. The dotted portion of II. corresponds to this. However, we do not think this could be the case, for that would mean that the potential at the bolometer end of the Lecher wires would be zero when the two parts of the bridge were 2 cm. apart whereas one would expect the potential still to be appreciable at 5 cm. just as we found it to be. Curves III. and III. *a* represent the potential in the neighborhood of a wire carrying a high frequency current, according to Abraham's theory. Considering our experimental arrangement it is perhaps too much to expect that Curves II. and III. should coincide. It is plain, however, from Curve II. that 80 per cent. of the magnetic

energy is to be found within a region of 1 cm. radius around the first part of the double bridge.

Now all this is in agreement with Lecher's experiment and with J. J. Thomson's[1] discussion of it. In other words we could say that the currents in the Lecher system are 100 per cent. transverse, for the potential at the ends of the circuit are the same whether the wires between the parts of the double bridge are cut or uncut. And yet we have had good success in treating the currents as strictly longitudinal, for we have verified Abraham's theory. Accordingly, we would like to emphasize this point. So long as the bridge is sufficiently short that the phase difference between its ends is inappreciable, it becomes *a matter of total indifference whether the currents be considered longitudinal or transverse.* For in such a case the induced current in the second half of the Lecher system neutralizes the primary current in the first half so far as the bridge itself is concerned and the two currents agree in phase. Just in proportion to the bridge length's becoming an appreciable fraction of the total length of the Lecher system so is this condition not fulfilled.

Effect of Several Bridges.

Early in the work we had tested the effect produced by the use of several bridges, but it soon became apparent that the usual method of fixing a bridge at the maximum nearest the oscillator belonging to a given tone and then exploring with a second bridge was not the proper way to work. To be sure many experimenters have found that the internodal distance is always shortened by the employment of additional bridges and indeed we agree with that. But in the light of our experiments it is clear that all exploring with additional bridges for odd-numbered harmonics should be made by starting with a fixed bridge at the free first and working in both directions. For the even numbered harmonics, the way we proceeded is illustrated in Fig. 13.[2] Having located by a single bridge the free seconds at 28.0 and 122.4, we fixed a bridge at 28.0 and explored with a second bridge the 122 region. The results are shown plotted to a large scale in the figure. The maximum

[1] Recent Researches, p. 466.

[2] The curve is that shown as Curve III., Fig. 12, of our former paper. It differs slightly from that however, in that the check receiver readings are taken into account. The oscillator was of such a length as to swing in with the free second. The maxima marked oscillator X. were found to respond to each other and were attributed to a second oscillator vibration due to close coupling. We think now there is some doubt about this. The free third which ought to occur at 12 is apparently combined with some oscillator vibration. At any rate it occurs at 15.5 instead of 12. What we have marked Oscillator X. and Y. could perhaps be due to the bolometer, but if so this would make the bolometer circuit selective. At present it seems safer to consider it non-selective until further work shows to what extent it is selective.

occurred at 121.7 instead of 122.4, and the readings were much larger than for a single bridge. Then with the second bridge fixed at 121.7 we reset the first bridge. The maximum was found at 28.7 instead of 28, the readings being 21 per cent. higher than for the free first or 45 per cent. higher than for the average free second taken with a single bridge.

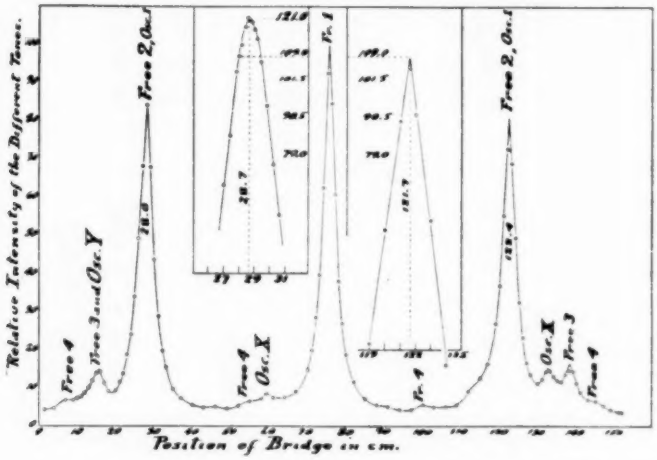

Fig. 13.

Second approximations as determined by further readjustments proved unnecessary. For instance, with a fixed internodal space of 93 cm. both bridges were moved at the same time by small steps but the readings were a maximum for the two positions above given. The internodal space for two bridges is thus 1.4 cm. shorter than for a single bridge or 1.5 per cent. less.

In a similar way, using a curve where the free thirds were prominent the internodal space was shorthened from 59.0 cm. to 58.1 cm. or again 1.5 per cent. Here again it was found best to work from the middle of the Lecher system outwards. That is, the middle bridge was left fixed and the other two changed. It ought to be remarked, however, that if the first bridge was left fixed and the second and third bridges changed the internodal space still remained practically 58.1 cm., although the readings were not quite so high as when the middle bridge was left fixed.

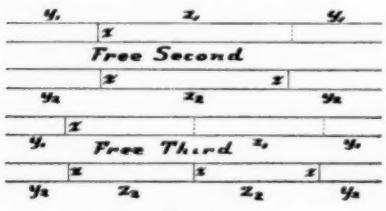

Fig. 14.

These data are extremely important in throwing light on several things. Consider the following diagram, Fig. 14. Fixing our attention

upon the free second harmonic it would seem from reasons of symmetry that the ratio

$$\frac{z_2 + y_2}{y_2},$$

considering the currents longitudinal, should equal

$$\frac{2(z_2 + y_2) + 3x}{2y_2 + x},$$

where the currents are taken as transverse. Since

$$y_2 = \frac{z_2}{2}$$

this reduces to an identity, as it should. Moreover, using first a single bridge, then two bridges, and considering the currents transverse it would seem fair for a first approximation to say

$$\frac{2(z_1 + y_1) + x}{2y_1 + x} = \frac{2(z_2 + y_2) + 3x}{2y_2 + x},$$

which when solved gives

$$y_2 - y_1 = \frac{x}{4}.$$

Similarly for the third harmonic we would have

$$\frac{4z_1 + 2y_1 + x}{2y_1 + x} = \frac{4z_2 + 2y_2 + 5x}{2y_2 + x}$$

whence

$$y_2 - y_1 = \frac{x}{3}.$$

Now x was 5 cm., whence

$$\frac{x}{3} = 1.67 \text{ cm.}$$

and

$$\frac{x}{4} = 1.25 \text{ cm.}$$

The values for $y_2 - y_1$ by experiment were 0.45 and 0.7 cm. respectively, our experimental figures being accurate to within a mm. in each case. Now Drude[1] in an important paper has considered theoretically the effect of a bridge upon the electromagnetic conditions present in the Lecher wires. He states that a part of the energy that reaches the bridge is reflected, a part goes past it and the rest through it. In general this is surely correct. Of course, the simple theory given by Abraham takes no account of any phase changes introduced by the presence of

[1] Wied. Ann., 60, p. 11, 1897.

one or more bridges. In general it would seem that y_1 does not exactly equal $z_1/2$. And yet in all our work so far we have assumed that $y_1 = z_1/2$, strictly. Now, we have seen in our previous work and in that of Blake and Ruppersberg that too close a coupling lengthens the internodal space of the higher tones so that they are no longer harmonic. On the other hand too loose a coupling shortens the internodal space for the higher tones and they are again not harmonic. And now we see there may be the possibility of an inharmonic relationship amongst the tones present because of a lack of symmetry of the two circuits into which the Lecher system is divided when only a single bridge is used. Of course, in the experiments reported in our earlier papers wherein we verified the Kirchhoff-Abraham generalization of the Thomson formula, it could easily have been that two of these factors so held each other in equilibrium as to make the verification possible. If this is true it means that the ideal case is to use a coupling close enough that the apparent displacement due to the coupling when a single bridge is used is just offset by the shortening of the internodal space when as many bridges are used as is the number of the harmonic under investigation.

In fact it would appear that in utter strictness Abraham's theory applies only for a bridge of zero length. As a *first approximation* it does very well for bridge-lengths up to 5 cm. but it will not do for second approximations. For the latter we are forced to accept Drude's[1] theory, rather. And in doing so we find, of course, that the shortening of the internodal space affords a satisfactory means of measuring the damping of the waves.

Using Drude's rotation, the "bridge-shortening" δb should equal $\frac{1}{2}l\zeta$, where l is the bridge-length and ζ is the ratio of the inductance of the bridge to that of one of the parallel wires per unit length. This also equals $\phi(\lambda/2)$, where λ is the wave-length of the harmonic under consideration and ϕ and ρ are two auxiliary quantities defined by the equations

$$\tan 2\pi\phi = \frac{2\pi\zeta\frac{l}{\lambda}}{1 - \gamma\zeta\frac{l}{\lambda}}$$

and

$$\rho^2 e^{2\gamma\phi} = \frac{1}{\left(1 - \gamma\zeta\frac{l}{\lambda}\right)^2 + \left(2\pi\zeta\frac{l}{\lambda}\right)^2}.$$

In these equations γ is the logarithmic decrement. Using the second

[1] Wied. Ann., 60, p. 1, 1897.

harmonic, we have

$$\phi = \frac{0.7}{93.0} = 0.00753,$$

whence $\zeta = 0.28$. For the third harmonic

$$\phi = \frac{0.45}{58.1} = 0.00774$$

and $\zeta = 0.18$. Transforming the first of these equations we have

$$\gamma = \frac{\tan 2\pi\phi - \frac{2\pi\zeta l}{\lambda}}{\tan 2\pi\phi \cdot \frac{\zeta l}{\lambda}}$$

and since

$$\phi = \frac{\zeta l}{\lambda}$$

very approximately,

$$\gamma = \frac{\tan 2\pi\phi - 2\pi\phi}{\tan 2\pi\phi \cdot \phi}.$$

Inserting the above values for ϕ for the second harmonic $\gamma = 0.100$, while for the third it is 0.103, a good agreement. The second of the equations above gives as an upper limit for γ the value 0.148. In interpreting the circuit for which we have thus measured the damping it should be remembered that we have measured the internodal space shortening for harmonics made abnormally high through resonance with the oscillator. Since in the Lecher circuit there is nothing to dissipate the electromagnetic energy, we must conclude that the logarithmic decrement for the *oscillator* was 0.102. We thus see that a very convenient method for measuring the damping of such an oscillator circuit as we have used is to let it resonate with a certain overtone of the Lecher system and measure the internodal space shortening, *but in the manner suggested above.* It is worth remarking that the "bridge shortening" is in our experiments far less than the half-bridge length, the value Drude roughly guesses at.

Drude says the "bridge shortening" can be quite appreciable without the amplitude of the wave reflected at the bridge becoming appreciable and this we agree with entirely. For the second harmonic, for instance, the ratio of the amplitudes of the reflected and incident waves comes out 0.9986. Similarly the ratio of the amplitudes of the current that passes beyond the bridge to the incident current is 0.0469. Squaring these numbers we have 0.9972 and 0.0022 which when added give 0.9994, a number which is practically equal to unity. It is well to be clear on the

interpretation of this. The current that gets past the bridge due to the incident current is very little. On the other hand, there is a large current through the bridge. This bridge-current sets up an induced current in the Lecher wires beyond the bridge. This induced current passing through the bridge in a direction opposite to the original bridge-current nullifies it almost entirely. The induced current plus the original current past the bridge make the current behind (away from the oscillator) the bridge practically equal to that before it and to the first order of approximation the bridge current is itself zero. In short, to the first order of approximation, the current in the Lecher system is *longitudinal*. Drude did not concern himself in his theory with the induced currents. We thus see that for a second and closer approximation to the facts the phase changes introduced by the presence of a bridge must be taken into account.

Further experiments along lines similar to those here presented are in progress.

Results of Other Investigators.

In our earlier paper the work of some other experimenters was referred to, but for the sake of comparison it seemed better to present our own experimental work before discussing their results. We limit ourselves to the work of those investigators who touch our work pretty closely.

Cohn and Heerwagen,[1] twenty-one years ago wrote a paper in which they not only developed the generalized Thomson formula but practically verified it to a rough approximation for all tones present, viz., the first six overtones. However, they state that "the series of overtones is not an harmonic one; the internodal space is smaller than the half wave-length and in general is related to it irrationally." Their work was done with a set of Lecher wires ending in the plates of a condenser. A Geissler tube constituted their detector. By changing the distance between the plates of the end-condenser they changed the Lecher system from a closed to an open circuit and so studied the phase-changes thereby introduced. When the wires end freely they state that there is a loop of potential at their ends and that their results fit their calculations as closely as in the corresponding case in acoustics of the open organ-pipe—in other words there is an end correction. We have shown that the amount of this correction is four per cent. for the experimental conditions that we have investigated.

These authors state that their results are in fair agreement with the generalized Thomson formula if the oscillator and Lecher circuits be considered a single circuit. This is, of course, the same consideration

[1] Wied. Ann., 43, p. 343, 1891.

into which we have been forced to explain some of the minor maxima that we have obtained.

Drude[1] in his critical discussion of the mode of action of a Lecher system seems to think that the Lecher and oscillator circuits can always be considered as a single circuit. This plainly is not the case. He, too, obtained inharmonic relations amongst the various overtones.

Lecher[2] himself and Drude as well were aware that exploration along a system of parallel wires by means of a single bridge gave a complicated system of waves, and so Drude suggests that the relations are much simpler if two bridges are used. This no one can question. He states, however, that no matter where the first bridge is placed, the second bridge will always determine the period of the oscillator. This we are inclined to question. Considering the possibilities of interaction between the oscillator and Lecher circuits and between the Lecher and receiver circuits[3] it seems best to accept the wave system whatever it is and then try to ferret it out and to investigate under what conditions it can be made simpler. It is this that we have attempted to do.

In the light of our experiments we have attempted to assign the various maxima given in the curves of Apt's[4] paper to definite tones of the Lecher system or of the oscillator. Unquestionably most of the maxima represent the free vibrations of the Lecher circuit. The maxima of his Fig. 5 are not all easily explained. Maximum 6 is the free first, 3 and 8 are the free seconds, 1, 6 and 9 the free thirds, 5 and 7 are presumably the free fourths with two prominent maxima 2 and 4 unexplained. For his curve *a*, Fig. 6, the free thirds are at 0, 180 and 360; the free seconds at 25 and 310; the free fourths at 145 and 250 presumably, with no maxima unexplained. For all of his figures, however, even a cursory glance shows how much out of position some of the maxima are. Doubtless this displacement of the maxima is due partly at least to lack of air-gaps and water-resistance in the oscillator circuit. The relative intensities of the different harmonics are far from ideal in all cases.

One thing in Apt's paper seems entirely wrong. In discussing his Fig. 6 he says circuit P is in resonance with circuit S when the spark gap is 5 cm. from the primary plates. Correspondingly R_1 is in resonance with R_2 when the Lecher bridge is 5 cm. from the secondary plates. One ought to expect in Fig. 6 an unusually strong maximum at 5 cm. But it doesn't appear. According to our view-point the best resonance

[1] Wied. Ann., 61, p. 631, 1897.

[2] Wied. Ann., 41, p. 850, 1890.

[3] Of course, if a Geissler tube is used as detector this last interaction would have no meaning. But such a detector at best is qualitative.

[4] Wied. Ann., 61, p. 293, 1897.

ought to exist for all four circuits when the spark gaps and the bridge are 180 cm. from their respective capacity plates. Had the large waves from the induction coil not swamped in part the free waves of the Lecher system the maximum at 185 in his Fig. 5 would have been the strongest of all.

SUMMARY.

We have shown in this paper that both forced and free oscillations can exist simultaneously upon the Lecher system and that the overtones of the oscillator are related to each other as the odd numbers, while the free overtones of the Lecher system are related to the fundamental as the natural numbers. When the two circuits are in resonance only the odd harmonics are present to any appreciable extent.

The influence of the receiver upon the wave-system was studied, both "selective" and "non-selective" receivers being used. When the oscillator and Lecher circuits were in tune with each other and with the first overtone of the receiver the ratio of "loop-strength" to "node-strength"[1] was found to be 325 to 1.

It was found that under certain conditions the Lecher circuit and either of the other circuits behaved in part as a single circuit, and in one case all three circuits vibrated as a single circuit (Figs. 3 and 4).

It has been shown that to the first order of approximation the currents in the Lecher system can be considered either as longitudinal or transverse, the bridge-current being practically non-inductive. To the same degree of approximation Abraham's theory can be said to hold. For the second order of approximation Drude's theory of standing waves explains the facts better, though an entirely satisfactory theory must consider the induced currents in the second half of the Lecher system. By applying Drude's theory the coefficient of damping of the oscillator was measured, the method being the very convenient one of measuring the internodal space shortening.

PHYSICAL LABORATORY,
OHIO STATE UNIVERSITY.

[1] These terms are due to Drude, l. c.

NUTATION IN PRACTICAL APPLICATIONS OF GYRO-ACTION.

By Burt L. Newkirk.

IN the discussion of the phenomenon of nutation presented by Professor Franklin in the January number of this Review the couple that produces the nutation is assumed to be applied suddenly. In the practical application of gyro-action it is important to take account of the fact that, owing to the non-rigidity of materials, a couple cannot be applied instantly, and to inquire what occurs when the applied couple increases from zero at a given rate. I prefer to treat the problem as follows:

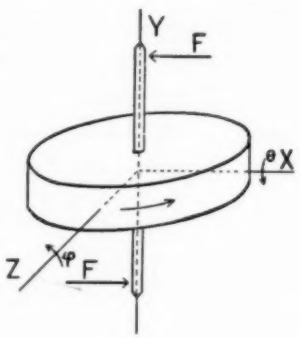

Fig. 1.

Suppose a flywheel of moment of inertia I about its axle, to rotate with angular velocity ω. A coördinate system being taken as shown in the figure, and rotation about the X and Z axes being measured by the angles θ and φ respectively, we assume a couple indicated by the forces FF to increase from zero uniformly with the time. The magnitude of this couple is represented by kt. The resulting motion of the flywheel is expressed by the equations

$$(1) \qquad \begin{aligned} j\frac{d^2\varphi}{dt^2} &= kt - I\omega\frac{d\theta}{dt}, \\ j\frac{d^2\theta}{dt^2} &= I\omega\frac{d\varphi}{dt}, \end{aligned}$$

where j is the moment of inertia of the flywheel and axle about a diameter of the wheel. These equations are rigorously correct for $t = 0$ and approximately so for a short time thereafter. By having the coördinate system rotate about the X axis at a suitable rate the equations can be given a form that will represent the motion for a longer time, but the form used here is entirely satisfactory for the present purpose, since we propose to study the motion for the short time only during which a couple is being brought to bear. We assume that during this short period of time the flywheel does not move far from the position shown in the figure.

The last term in each equation expresses the effect of "gyroscopic resistance."[1]

The solution of these equations is of the form

$$\begin{aligned} \varphi &= a \sin \lambda t + b \cos \lambda t + ct^2 + dt + e, \\ \theta &= a' \sin \lambda t + b' \cos \lambda t + c't^2 + d't + e'. \end{aligned} \tag{2}$$

The initial conditions are:

$$\varphi = 0, \qquad \theta = 0,$$
$$\frac{d\varphi}{dt} = 0, \qquad \frac{d\theta}{dt} = 0,$$
$$\frac{d^2\varphi}{dt^2} = 0, \qquad \frac{d^2\varphi}{dt^2} = 0.$$

The remaining relations used in determining the constants are obtained by substituting expression (2) for φ and θ in (1), remembering that the resulting equations must be satisfied for all values of t so that absolute term, coefficient of t and coefficients of $\sin \lambda t$ and $\cos \lambda t$ must be separately equal to zero. We find

$$a = -b' = e' = -\frac{kj^2}{(I\omega)^3},$$
$$d = \frac{kj}{(I\omega)^2},$$
$$c' = \frac{k}{2I\omega},$$
$$\lambda = \frac{I\omega}{j},$$

the remaining constants of integration being zero, so that the solutions are:

$$\varphi = -\frac{kj^2}{(I\omega)^3} \sin \frac{I\omega}{j} t + \frac{kj}{(I\omega)^2} t,$$
$$\theta = \frac{kj^2}{(I\omega)^3} \cos \frac{I\omega}{j} t + \frac{k}{2I\omega} t^2 - \frac{kI\omega}{j^2}.$$

The period of the vibration is

$$T = 2\pi \frac{j}{I\omega}$$

and the amplitude

$$\frac{kj^2}{(I\omega)^3}.$$

[1] See Spinning Tops, etc., by H. Crabtree, page 44, or Theorie des Kreisels, by Klein and Sommerfeld, Vol. IV., § 1.

If in place of the couple kt we have a couple C suddenly applied and constant in magnitude the solution of the equations of motion would be

$$\varphi = -\frac{Cj}{(I\omega)^2}\cos\frac{I\omega}{j}t + \frac{Cj}{(I\omega)^2},$$

$$\theta = -\frac{Cj}{(I\omega)^2}\sin\frac{I\omega}{j}t + \frac{C}{I\omega}t.$$

The period of the vibration is the same as before, but the amplitude is in general much larger since the large quantity ω which appears in the denominator is here raised to the second power only, whereas in the other case it appeared raised to the third power. To examine this point more carefully let us compare the effect of a suddenly applied couple C with the effect that would be produced if the same couple were to grow from zero to its full magnitude in τ seconds. Denoting the amplitude of the resulting vibrations by A and A' respectively we have, since $\tau k = C$,

$$A = \frac{kj^2}{(I\omega)^3} = \frac{Cj^2}{\tau(I\omega)^3}, \quad A' = \frac{Cj}{(I\omega)^2},$$

$$\frac{A}{A'} = \frac{j}{\tau I\omega} = \frac{1}{2\tau\omega},$$

if we set $j = \frac{1}{2}I$, as would be the case if the wheel were a thin disc and the mass of the axle negligible. Since the angular velocity of the flywheel (ω) is very large in the case of the stabilizing apparatus of the monorail car for example, and the important disturbing couples are not likely to come in the form of sharp blows the fraction A/A' will be very small, and the solution which assumes the couple to increase with the time will give a result much more in accordance with the facts than the solution based upon the ordinary theory of nutation, which assumes the couple to be instantly applied with its full force.

As a numerical example let us suppose the flywheel used to stabilize a monorail car to be a disk of nickel steel 4 feet in diameter rotating at the rate of 4,000 R.P.M. We have then the approximate value

$$\frac{A}{A'} = \frac{1}{2\tau\omega} = \frac{1}{838\tau},$$

which would be a small quantity even if τ were as small as $0^s.01$.

Computations of the amplitude and period of the nutational vibration for various assumed cases indicate that it will have to be given attention in the design of the monorail car, and especially in connection with the control device, but that it will probably not present any insurmountable obstacle to the development of a gyro-stabilizing system for cars.

When the impulse I of the flywheel is ample to satisfy the other requirements of the problem the nutation will be of very small amplitude and short period, so that it may be readily destroyed by damping. In the case of the monorail car the equations are less simple than those discusssed above owing to the fact that the car upon which the flywheel is mounted is free to rotate about the rail, and is in unstable equilibrium. The solutions show however that the amplitude of nutation will be much less than the value corresponding to sudden application of the disturbing couple. The expression for the amplitude of the nutation when the load is applied gradually (as must be the case due to the non-rigidity of materials) contains in the denominator the quantity ω to a power one unit higher than the power of this quantity that occurs in the corresponding expression under the supposition of instantaneous application of the disturbing force.

CIRCULAR DICHROISM AND ROTATORY DISPERSION OF CERTAIN SALT SOLUTIONS.

By L. B. Olmstead.

THE object of this investigation was to study the rotation and ellipticity produced in plane polarized light by transmission through solutions containing an optically active acid radical combined with an absorbing metallic ion; and, in connection with this, the refractive indices and extinction coefficients of the same solution.

These solutions are interesting because of the discovery by Cotton[1] that certain of them show, in addition to the rotation of the plane of polarization, "circular dichroism"—that is, they absorb unequally right and left circularly polarized light, and therefore convert plane polarized light into elliptic. In all the solutions in which the latter effect is found both the ellipticity and the rotation produced are anomalous.

The work of Cotton, which was confined to copper and chromium tartrate solutions, was repeated and extended by McDowell,[2] who found the effect also in potassium cobalt tartrate, nickel tartrate and copper malate. Although the method used by McDowell allowed him to make observations at all parts of the spectrum, while Cotton had been limited to a few definite wave-lengths, the formulæ he used for computing ellipticities were incorrect and hence his results only qualitative.

This subject has been treated theoretically by Natanson[3] for the case of mono-electronic substances—substances showing a single absorption band. He finds a very satisfactory agreement with Cotton's observations on chromium potassium tartrate but calls attention at the same time to the insufficiency of the mono-electronic theory for the majority of substances investigated.

Apparatus.

For Measuring Ellipticities.—The Brace half-shade system was used for this purpose. The optical arrangement is shown in Fig. 1. Strong sunlight condensed by the lens L_1 on the slit S_1 of a spectral instrument was separated by the latter into a spectrum in the plane of the slit S_2.

[1] Ann. de Chim. et de Phys., S. 7, T. 8, p. 347, 1896.

[2] Phys. Rev., 20, p. 163, 1905.

[3] Natanson, Ladislas, Bull. de l'Acad. des Sciences de Cracovie, Oct., 1908, Jan., 1909.

Through this slit a band of "monochromatic" light passed whose extreme wave-lengths—differing by not more than 20$\mu\mu$—depended on the width of the two slits. The lack of homogeneity of this light introduced an error in the measured ellipticity which was appreciable only at those points where there is a rapid change of direction of the ellipticity curve. Adjustments were made to reduce this error to about the same magntiude as the others inherent in the method. From S_2 the light passed successively through a condensing lens L_2, the polarizer P, the tested solution R_2, to the analyzing system consisting of half-shade H,

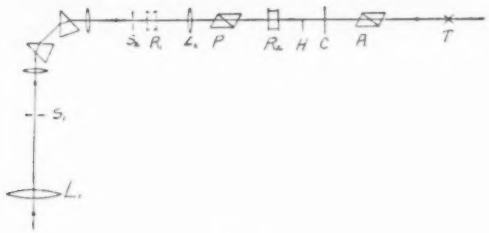

Fig. 1.

compensator C, analyzer A, and the telescope T focused on the edge of the half-shade.

The half-shade was used with its principal azimuth at 45° to that of the analyzer. The compensator was mounted on a circle graduated to measure to .01°.

A measurement consisted in removing R_2 and placing an equal cell R_1 of the same solution before the polarizer, then adjusting the polarizer for minimum intensity of field and the compensator for a "match." This setting on plane polarized light gave the "zero" position of the compensator. The cell R_1 was then removed, R_2 inserted between polarizer and half-shade and the adjustment of polarizer and compensator carried out as before. From this rotation (φ) of the compensator the ellipticity of the light was computed—the initial setting having been made on light of zero ellipticity. The object of the auxiliary cell R_1 was to furnish the same spectral distribution of light for the zero setting as for that on the ellipticity. The rotation of the polarizer was to compensate for the rotation of the light produced by the solution. It was accomplished with sufficient accuracy by setting for minimum intensity of field.

The compensator was calibrated by Mr. Tate, of this laboratory, using Tool's[1] method. Its order was 7.56° for 550$\mu\mu$. The value for other wave-lengths was obtained from Tool's dispersion curve for mica.

[1] Tool, A. Q., Phys. Rev., XXXI., No. 1, p. 1, 1910.

For calculating the ellipticity Tuckerman[1] has shown that

$$E = \sin 2\theta tg\pi N (\sin 2\psi - \sin 2\psi_0),$$

where θ is the angle between the major axis of the incident light and the principal azimuth of the half-shade, N is one half the order of the compensator, ψ is the angle between the principal axis of the compensator and the nicol for a match on light of the given ellipticity, ψ_0 the same for plane polarized light, and E is the ratio of the minor to the major axis of the ellipse. Since the half shade was set with its principal azimuth at 45° to the polarizer, $\sin 2\theta = 1$, and the formula reduces to the form

$$E = tg\pi N[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0],$$

where $\varphi = -(\psi - \psi_0)$, and $\varphi_0 = \psi_0$. φ is obtained by rotating the compensator from a match on plane polarized light to a match on light of ellipticity E, and φ_0 by making complementary settings on plane polarized light, then subtracting half their difference from 45°. With the analyzing system used φ_0 was 6°.5 for all wave-lengths.

For Measuring Rotation.—The half-shade and analyzer were removed and the compensator replaced by a Lippich analyzing nicol, on the edge of which the telescope was focused. The measurement of rotation was then carried out as for ellipticity.

For Measuring Absorption Coefficients.—For this a Brace spectrophotometer was used in the customary way, the effect of reflection being eliminated by employing two different lengths of the solution. The extinction coefficient (nk) per wave-length in the medium was calculated from the formula

$$nk = \frac{n\lambda}{4\pi(l_2 - l_1)} \log \frac{J_1}{J_2},$$

$n\lambda$ being the wave-length (*in vacuo*) of the light, n the refractive index of the solution, and J_1/J_2 the ratio of the intensities of the light after passing through solutions of lengths l_1 and l_2 centimeters respectively.

Accuracy.

The method used for measuring ellipticity is capable of greater accuracy than that reached by Cotton, but its principal advantage lies in its flexibility, in that readings may be made in any part of the spectrum. The rotation of the compensator could be read to .01°, and the probable error of the mean of any set of readings was not larger than .1°. The cells varied in length from .95 cm. to 4.40 cm., with ends of cover glasses selected free from double refraction. The values of the rotation as

[1] Tuckerman, L. B., Univ. of Neb. Stud., No. 2, 1909.

given are accurate to .02°; the extinction coefficients to 2 per cent.; and the refractive indices to 0.0003.

EXPERIMENTAL RESULTS.

The values of the rotation and ellipticity as given in the tables and curves were calculated from the observed values for *solutions one centimeter in length*. To bring all curves within the same plot the magnitude of their ordinates was changed as indicated on them. No chemical analyses of the compounds were made; the *names assigned being merely for convenience*, and not indicating that the chemical formulæ are known. The chemicals used throughout were c.p., and most of them obtained from Baker and Adamson or Kahlbaum. Most of the observations were made at a room temperature of 27° to 30° C.

Chromium Compounds.—Table I. gives values for selected wavelengths obtained from a chromium potassium tartrate solution made

TABLE I.

$K_2Cr_2O_7$ + Potassium Tartrate.

$\lambda(\mu\mu)$	Ellipt. $\times 10^3$.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	− 4.3	− .73	—	——
470	3.4	− .45	4.7	1.3412
510	− 2.8	− .75	3.3	1.3391
550	−20.4	− .54	5.6	1.3374
570	−26.8	.36	7.6	1.3368
590	−21.7	.89	8.6	1.3361
610	− 6.0	1.30	8.7	1.3356
630	2.1	1.11	7.9	1.3349
650	5.5	.70	6.2	1.3344
690	4.8	.27	3.4	1.3336

according to Cotton. Two warm solutions, the one containing .750 gm. of potassium bichromate ($K_2Cr_2O_7$) and the other 10.00 gm. of neutral potassium tartrate ($K_2C_4H_4O_6$), were mixed and kept at a temperature of about 65° C. for several hours. The solution was then allowed to evaporate to dryness, after which the excess potassium tartrate was removed by fractional solution, fractional crystallization, and mechanical separation, leaving only green-colored crystals. This material was dried over P_2O_5, and then dissolved in distilled water, the solution being made accurately in the ratio 1 : 30 by weight.

This solution gave ellipticity which has a maximum negative value at 575$\mu\mu$, just to the left of the region of maximum absorption, which is in the yellow. (See curves plotted from the complete series of observa-

tions, Fig. 2.) The rotation becomes zero near the same point (at 565$\mu\mu$), remaining positive for the longer wave-lengths and negative for the shorter. The change in direction of ellipticity and the increasing negative rotation in the violet are probably due to the absorption band indicated in that region by the curve for nk. The black circles and broken lines give the values obtained by Cotton, as calculated for the concentration used here. The agreement is very satisfactory.

By using 1.20 gm. of potassium bichromate instead of .75 gm., the crystalline mass was practically free from excess potassium tartrate, and the solution found to give nearly the same values.

The procedure for obtaining all the chromium compounds was much the same as that outlined above. When solutions of chromic acid anhydride (CrO_3) and neutral potassium tartrate are mixed and kept at

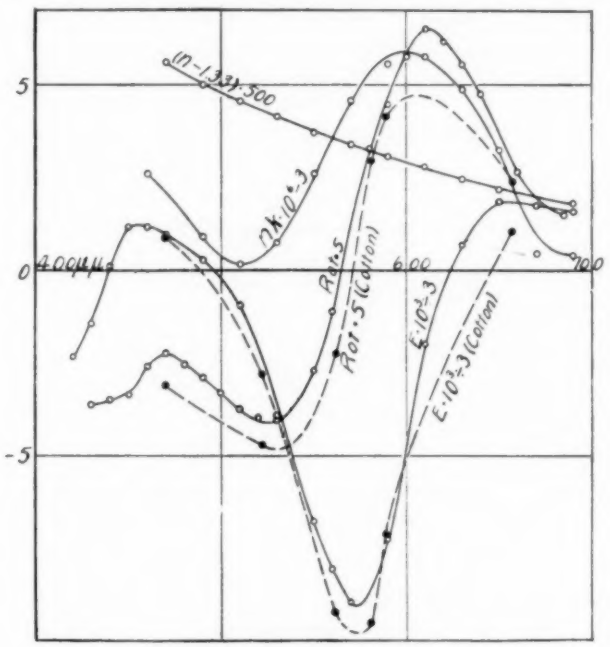

Fig. 2.

$K_2Cr_2O_7$ + potassium tartrate.

30° C. for half an hour the reaction is complete. The results in Table II. are for 1 gr. of CrO_3 to 15 gr. of potassium tartrate, the final solution being made 1 : 40. The curves (Fig. 3) are found to be very similar in form and magnitude to those obtained from the preceding solution, which was 1 : 30. In the same figure the refractive index curve of distilled water is also plotted. A comparison of this with the corresponding curve

TABLE II.

CrO_3 + Potassium Tartrate.

λ(μμ)	Ellipt. × 10³.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	2.6	−.19	6.0	——
470	1.4	−.49	4.5	1.3402
510	− 5.0	−.72	2.9	1.3381
550	−19.1	−.64	5.1	1.3365
570	−27.0	−.09	7.1	1.3358
590	−24.6	.55	7.7	1.3351
610	−11.9	.92	7.2	1.3345
630	− 2.5	.89	6.7	1.3339
650	2.9	.59	5.3	1.3333
690	3.5	.23	2.9	1.3322

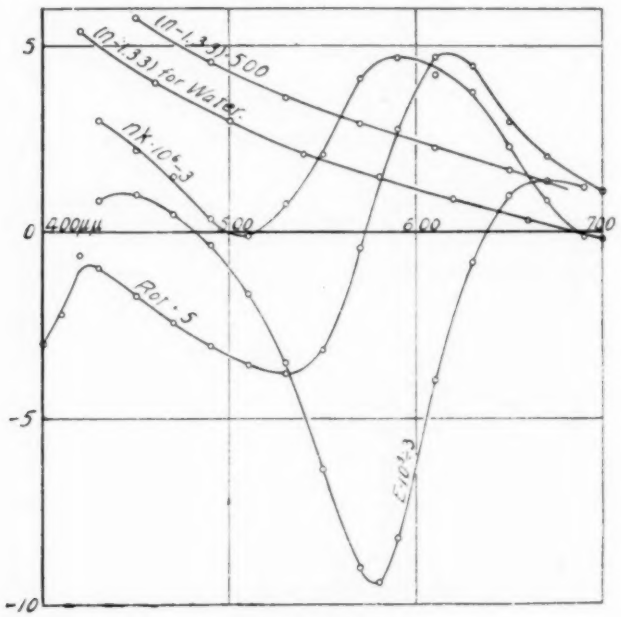

Fig. 3.

CrO_3 + potassium tartrate.

for the solution shows no trace of any anomaly, though the form of all the other curves on the plot would suggest its presence. In fact in all the solutions tested the anomaly in the refractive index curve, which theoretically should be present, is too small to be detected.

Observations made on the same stock solutions at different times indicated that the above compounds are stable.

Table III. and Fig. 4 (full lines) give the results obtained from chro-

TABLE III.

$Cr(C_2H_3O_2)_3$ + Potassium Tartrate.

λ(μμ)	Ellipt. × 10^5.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	−1.09	.42	—	——
460	− .45	.42	5.8	1.3408
490	.36	.46	4.4	1.3391
520	1.80	.52	7.6	1.3377
550	2.87	.48	13.3	1.3366
570	3.56	.37	14.4	1.3359
600	2.12	−.07	12.5	1.3346
630	− .17	−.27	8.4	1.3333
660	− .58	−.19	5.1	1.3328
690	− .50	.09	3.2	1.3322

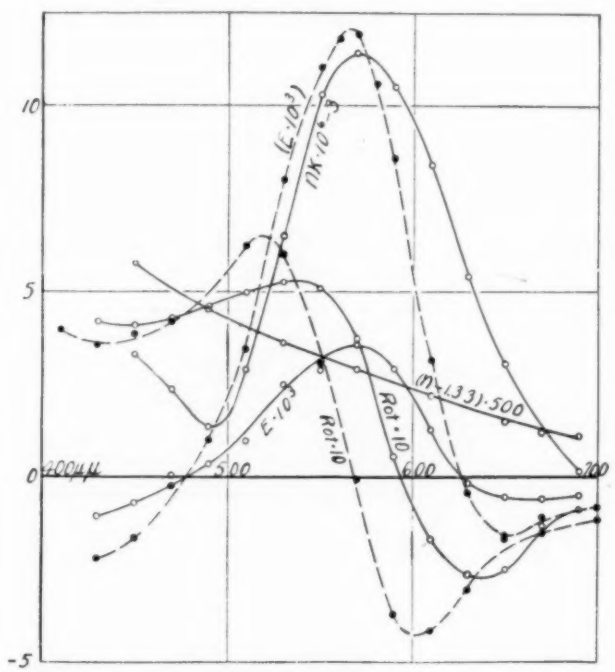

Fig. 4.

Chromium acetate + potassium tartrate.

mium acetate $Cr(C_2H_3O_2)_3$ and neutral potassium tartrate (1 : 8) dissolved in water (1 : 30). The ellipticity and rotation are opposite in sign to the other chromium solutions. This solution was also more strongly absorbing. The black circles and broken lines, representing the values of the rotation and ellipticity obtained from the same stock solution five weeks later, reveal the probable instability of the compound, although

the two sets of ellipticity curves have their maxima in the same region and pass through the zero value at the same wave-lengths. The rotation curves are however relatively displaced. The difference in the magnitude of the ellipticities is especially striking.

Table IV. and Fig. 5 give the results from a chromium potassium

TABLE IV.

CrO_3 + Potassium Lactate.

$\lambda(\mu\mu)$	Ellipt. $\times 10^3$.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	− .68	−.13	7.4	——
460	.47	−.10	6.7	1.3425
490	.24	−.12	5.2	1.3406
520	−1.02	−.13	5.7	1.3390
550	−2.71	−.07	9.1	1.3378
570	−3.30	−.01	10.7	1.3372
600	−2.52	.06	11.1	1.3362
630	−1.22	.09	9.5	1.3353
660	− .60	.06	6.6	1.3345
690	− .40	.05	4.4	1.3338

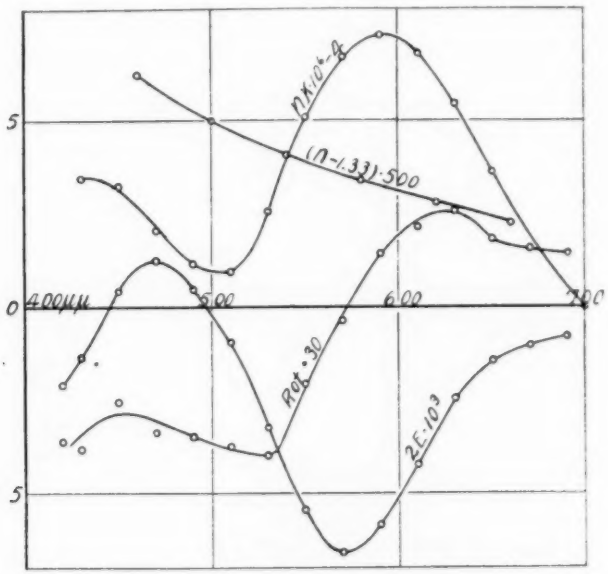

Fig. 5.

CrO_3 + potassium lactate.

lactate produced by the reaction of CrO_3 with potassium lactate (1 : 16), dilution 1 : 30. This compound was the only lactate obtained which gave circular dichroism; although trials were made with salts of copper, cobalt, manganese, nickel, iron, and uranium.

It seems likely that the above compounds are double chromium potassium tartrate salts. That the potassium (or sodium, or ammonium) is not a necessary constituent however is shown by the reaction of CrO_3 on tartaric acid. Table V. and Fig. 6 give the results from the crystal

TABLE V.

CrO_3 + Tartaric Acid.

λ(μμ)	Ellipt. × 10^3.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	− .71	.18	5.1	——
460	− .45	.17	4.3	1.3422
490	− .51	.14	3.8	1.3405
520	.78	.17	7.1	1.3392
550	.21	.20	10.8	1.3381
570	1.52	.17	10.9	1.3373
590	2.22	.10	9.3	1.3367
620	1.79	.06	6.0	1.3358
660	.99	.02	2.7	1.3347
700	.04	.01	1.4	1.3336

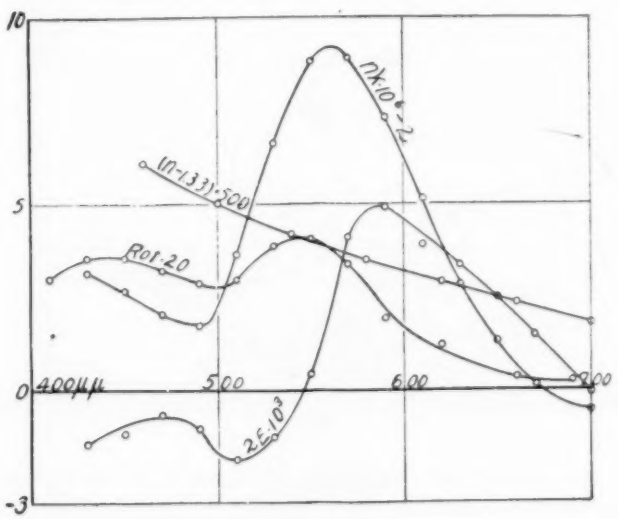

Fig. 6.

CrO_3 + tartaric acid.

formed from this reaction—1 gm. of CrO_3 on 15 gm. of tartaric acid (1 : 15), dissolved in distilled water in the ratio 1 : 30. The rotation is anomalous in form, but positive in value for all wave-lengths.

Table VI. and Fig. 7 give the values obtained from a chromium malate compound, made by the reaction of CrO_3 on malic acid (1 : 15), dilution

TABLE VI.

CrO_3 + Malic Acid.

λ(μμ)	Ellipt. × 10³.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	.34	—	4.9	——
460	.21	−.04	2.7	1.3418
490	.09	−.04	2.7	1.3401
520	− .24	−.06	5.8	1.3388
550	−1.06	−.03	10.0	1.3377
570	−1.13	.02	10.5	1.3370
590	− .68	.04	9.4	1.3364
620	− .06	.01	5.7	1.3354
660	− .08	.01	2.8	1.3343
700	− .03	.01	1.3	1.3333

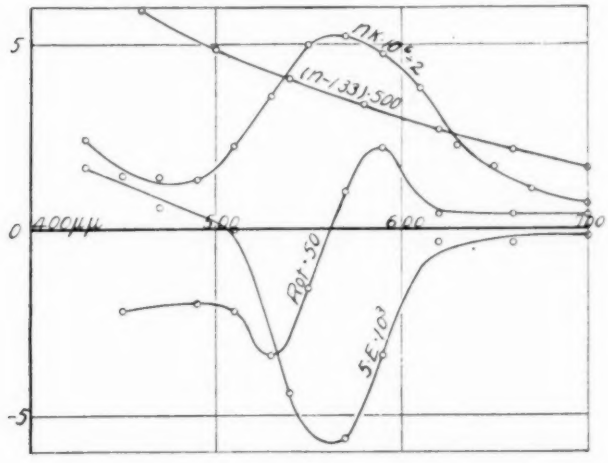

Fig. 7.

CrO_3 + malic acid.

1 : 30. The magnitude of both the rotation and the ellipticity is relatively small.

Solutions giving circular dichroism were made, using potassium chromate (K_2CrO_4), chrome alum, and chromium nitrate. Instead of potassium tartrate one may use sodium or ammonium tartrate, or Rochelle salt, or cream of tartar. The results from these were so similar to the ones given above that they are of no special interest. Reaction of CrO_3 with several other organic compounds was obtained, but none of them gave circular dichroism.

A glance at the results from the various chromium compounds shows that all the extinction coefficient curves are similar in form; the rotation is anomalous—in every case but one reverses sign in passing through

the absorption band; and the maximum ellipticity is shifted slightly from the position of maximum extinction.

Copper Compounds.—Cotton made a copper tartrate solution by dissolving 1 gm. of copper tartrate in 100 c.c. of water containing 1 gm. of KOH. The difficulty with this solution is that CuO is slowly precipitated. A similar solution was made here by using 1.50 gm. of KOH, 2.00 gm. of copper chloride, and 10.00 gm. of neutral potassium tartrate; which has the advantage in that no CuO is formed. Table VII. and Fig. 8 give the results obtained from this solution. All magnitudes are

TABLE VII.

$CuCl_2 + K_2C_4H_4O_6 + KOH.$

λ(μμ)	Ellipt. × 10⁵.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	.0	.42	.4	1.3442
470	.1	.43	.3	1.3416
510	.2	.44	.7	1.3396
550	1.6	.49	2.2	1.3380
590	4.5	.55	5.1	1.3367
620	7.9	.39	8.1	1.3358
640	8.8	.35	9.4	1.3352
660	9.7	.35	10.9	1.3348
680	9.0	.38	11.7	1.3344

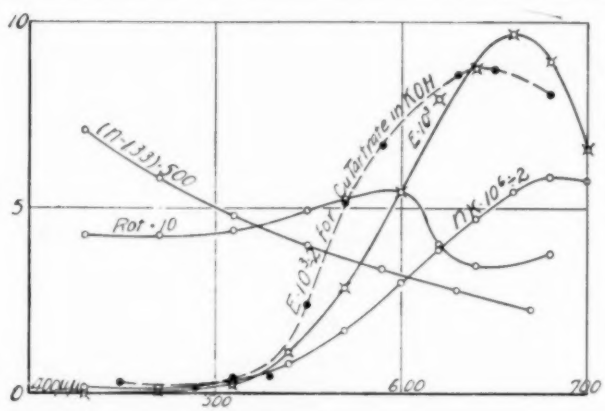

Fig. 8.

$CuCl_2 + K_2C_4H_4O_6 + KOH.$

positive. The absorption is maximum at 680μμ, and the ellipticity at 660μμ, both decreasing toward the violet. The black circles and broken line curve is plotted from values of the ellipticity obtained from a copper tartrate solution made according to Cotton's method. The similarity is evident.

Table VIII. and Fig. 9 give the results obtained from a copper malate solution made by saturating a solution of malic acid at 20° with freshly precipitated washed copper hydroxid, evaporating the solution, and dissolving the dry material in water (1 : 30). This solution shows a strong

TABLE VIII.

Copper Malate.

λ(μμ)	Ellipt × 10³.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	—	−.20	.3	1.3438
470	.12	−.17	.1	1.3415
510	.12	−.18	.3	1.3398
550	.09	−.19	1.4	1.3382
590	− .28	−.21	4.2	1.3368
620	− .82	−.25	7.8	1.3360
640	−2.03	−.28	11.2	1.3355
660	−3.00	−.27	14.8	——
680	−1.24	—	15.9	——

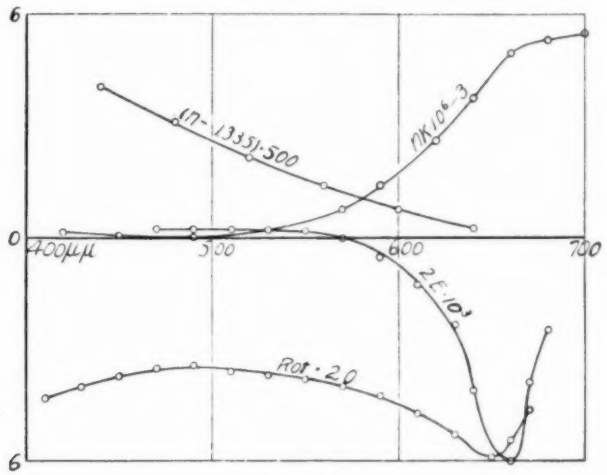

Fig. 9.

Copper malate.

absorption band in the red. The ellipticity changes through zero from a small positive magnitude in the blue and green to a sharp negative maximum in the red, at 660μμ. The rotation which is slightly irregular in form is negative for all wave-lengths.

Cobalt Compounds.—A dichroic potassium ammonium cobalt tartrate compound was made according to McDowell's method. A solution of cobalt chloride and neutral potassium tartrate (1 : 4) was boiled for 15 minutes and then evaporated to dryness. The red crystals were sepa-

rated out by fractional solution, and, after recrystallizing and drying, were dissolved in water (1 : 9) which contained a small amount of ammonia. This gave a wine-colored solution with an absorption band in the green. Table IX. and Fig. 10 give the results from this solution.

TABLE IX.

Cobalt Tartrate in Ammonia.

λ(μμ)	Ellipt. × 10^3.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	1.18	—	4.3	—
460	2.10	.65	5.2	1.3524
490	4.45	.59	6.6	1.3505
520	7.15	.48	8.4	1.3490
550	8.18	.13	9.4	1.3477
580	2.48	−.42	7.4	1.3467
610	−2.21	.05	4.4	1.3458
640	− .61	.26	2.9	1.3449
670	2.83	.25	2.4	1.3441
700	3.95	.07	2.4	1.3434

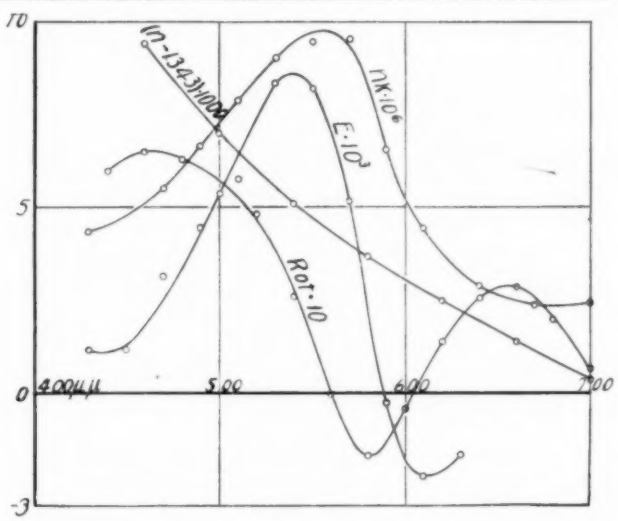

Fig. 10.

Cobalt tartrate in ammonia.

The ellipticity curve is similar in form to McDowell's, but the rotation is very different—probably owing to the fact that in the present case most of the excess potassium tartrate was removed.

Ellipticity and rotation of a different type were obtained by boiling a solution containing cobalt ammonium sulphate and ammonium tartrate (1 : 3) and dissolving in water (in the ratio 1 : 9) the red crystals so

obtained. Table X. and Fig. 11 show the results. The ellipticity which is very small reaches a maximum negative value at 500$\mu\mu$—being slightly displaced as in most of the cases, from the position of maximum absorption. The rotation is positive, and only slightly irregular in slope Tests

TABLE X.

$Co(NH_4)_2(SO_4)_2 + (NH_4)_2C_4H_4O_6$.

$\lambda(\mu\mu)$	Ellipt. $\times 10^3$.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	−.04	.45	.9	1.3553
460	−.13	.37	2.5	1.3530
490	−.64	.37	4.4	1.3514
520	−.52	.35	5.7	1.3499
550	−.20	.31	4.5	1.3486
580	.12	.27	2.1	1.3476
610	.16	.24	1.2	1.3466
640	.16	.21	.9	1.3457
670	.14	.18	.9	1.3449
700	.10	.16	1.1	1.3441

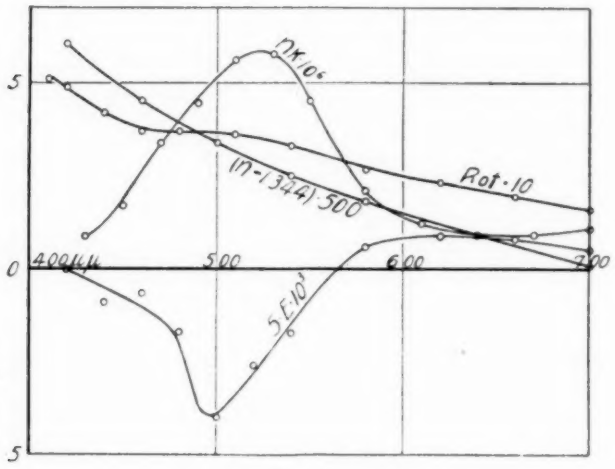

Fig. 11.

Cobalt ammonium sulfate + ammonium tartrate.

made later on the stock solutions showed that the ellipticity of both of these compounds decreased with time.

Manganese Compounds.—A saturated water solution (about 1 : 400) of manganese tartrate at about 40° was made, and to it was added a small quantity of a dilute solution of ammonia. The color changed from a very faint flesh color to red. An absorption band (see Fig. 12) unundoubtedly exists in the extreme violet and another slight one in the

TABLE XI.

Manganese Tartrate in Ammonia.

λ(μμ)	Ellipt. × 10³.	Rot. (Deg.).	Ext. Coeff. ($nk \times 10^6$).	Refr. Ind. (n).
430	− .80	.18	—	1.3404
460	1.10	.19	4.9	1.3385
490	2.00	.13	4.9	1.3368
520	2.28	.07	4.9	1.3355
550	2.35	.01	4.8	1.3343
580	1.87	−.04	4.6	1.3332
600	1.13	−.06	4.1	1.3326
630	.23	−.05	3.8	1.3318
660	− .35	−.03	3.7	1.3310
700	− .38	.00	3.6	1.3300

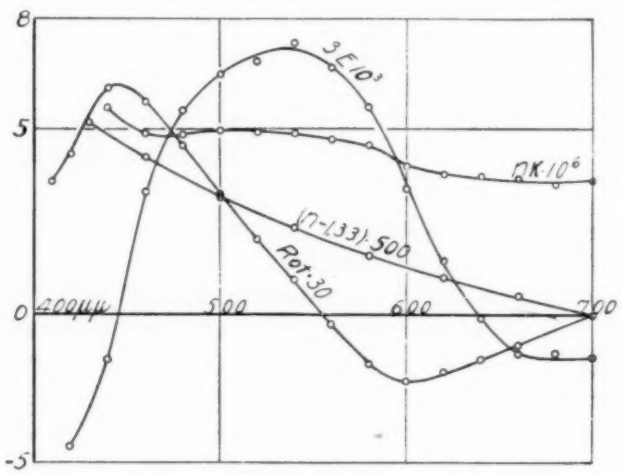

Fig. 12.

Manganese tartrate in ammonia.

green. The ellipticity changes rapidly from a negative value at 420μμ, through zero to a maximum positive value at 540μμ, decreasing again to a small negative value in the red. The effect is large for such low concentration. The rotation shows maximum positive and negative values respectively below and above 550μμ.

A similar solution was made using potassium hydrate instead of ammonia, but a red precipitate formed before observations could be made.

Attempts to obtain the nickel tartrate solution, reported by McDowell as showing circular dichroism, were unsuccessful.

No ellipticities were obtained from compounds containing nickel, iron, or uranium, although many solutions were tried.

To sum up, the colored compounds successfully used were chromium, copper, cobalt, and manganese; and the organic were tartrates, malates, and lactates, these being closely related optically active compounds.

In conclusion, the writer wishes to thank Professors Skinner and Tuckerman for the aid they have given him in this work.

BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA.

THE DEFINITION OF AN IDEAL GAS.

By E. F. Farnau.

Introductory.

THE ideal gas has been treated with more or less completeness in all the texts on thermodynamics and in a considerable number of articles bearing directly upon the subject. The matter is quite worthy of consideration in that the concept of an ideal gas permits of simple though approximate formulation of the characteristic equation for a one-component body in a limited region of one-phase states, forms a basis upon which to build a more accurate characteristic equation for a one-component body, affords a convenient hypothetical scale of temperatures, and admits of ready determination of the relation between this scale of temperatures and the thermodynamic scale.

Among the more recent papers may be mentioned the following: Bakker,[1] criticizing Poincaré's[2] text, has deduced a relation existing between the laws of Boyle, Gay-Lussac and Joule; Baynes[3] has indicated the consequences of these and other laws taken singly and two at a time; Carré[4] and Pellat[5] have discussed the laws of Gay-Lussac and their relation to the law of Joule; Webster and Rosanoff,[6] purposely disregarding the law of Gay-Lussac, have formulated the laws of Boyle, Joule and of the zero Joule-Thomson effect, showing their interdependence; finally, Buckingham,[7] in a monograph on the subject, has introduced the much-needed arbitrary scale of temperatures, and also the specific heat relations. None save the last-named author has endeavored to give a complete treatment of the subject.

The present paper is an attempt to discuss the matter from a somewhat different standpoint, in order to bring out more clearly the requirements for complete definition of the ideal gas.

[1] Bakker, Zeit. phys. Chem., *14*, 671 (1894); *17*, 171, 678 (1895); *20*, 461 (1896); *22*, 543, (1897); Jour. de Phys. (3), *7*, 152 (1898); *8*, 214 (1899); Jour. Chim. Phys., *4*, 67 (1906); Phys. Rev., *31*, 589 (1910).

[2] Poincaré, Thermodynamique.

[3] Baynes, Zeit. phys. Chem., *18*, 335 (1895); *21*, 556 (1896); Proc. Phys. Soc. Lond., *15*, 361.

[4] Carré, Jour. de Phys. (3), *7*, 718 (1898).

[5] Pellat, Jour. de Phys. (3), *8*, 100 (1899).

[6] Webster and Rosanoff, Phys. Rev., *29*, 304 (1909).

[7] Buckingham, Bull. Bur. Stands., *6*, No. 3 (1910).

THE LAW OF BOYLE.

Concerning the statement of this law no misunderstanding has arisen. It is given briefly as follows: The volume of unit mass of any gas at a fixed temperature is inversely proportional to the pressure. It may be formulated

$$pv = F(\tau), \tag{1}$$

where p is the pressure, v the volume of unit mass of the gas, and $F(\tau)$ some function of the temperature τ of an arbitrary temperature scale. τ is some uniform increasing or decreasing function of the degree of hotness of the body.

THE LAWS OF GAY-LUSSAC.

(*a*) *Formulation of the Laws of Gay-Lussac.*—Here considerable differences of opinion exist. The following are current statements of the laws: The pressure being constant, the volume of a unit mass of gas varies directly as its absolute temperature (the temperature being implicitly considered to be measured on the thermodynamic scale); and, the volume of a unit mass of gas being constant, the pressure varies as the absolute, *i. e.*, thermodynamic, temperature. It is obvious that historically at least there is no justification for such definitions. At the time of Gay-Lussac, the scale of temperatures was defined by assumption of the constancy, between the two fixed temperatures of melting ice and boiling water, of the volume-temperature derivative of some arbitrary substance —mercury, alcohol, or one of the "permanent" gases. That the thermodynamic scale of temperatures later defined by Kelvin should for a wide range agree so closely with that of the constant-pressure gas thermometer has nothing to do with the observations of Gay-Lussac.

A correct statement of Gay-Lussac's laws is: For constant pressure, the relative thermal expansion of a mass of gas between two assigned temperatures is a constant, which is independent of the constant pressure, and of the nature of the gas; and, for a constant volume of a mass of gas, the relative pressure increase between two assigned temperatures is a constant, which is independent of the constant volume, and of the nature of the gas. The two laws are formulated as follows:

$$\frac{v(p, \tau_1) - v(p, \tau_0)}{v(p, \tau_0)} = \alpha; \quad \frac{p(v, \tau_1) - p(v, \tau_0)}{p(v, \tau_0)} = \beta.$$

Gay-Lussac observed that α and β were approximately equal for the particular gases under the particular conditions of his experiments. It will be noted that equality of α and β really constitutes a third law of

Gay-Lussac, since this is altogether independent of the above statements of the two laws. However, in order not to complicate matters unnecessarily, this assumption shall in the following be implicitly made.

(*b*) *Definition of the Scale of Temperatures of the Constant-pressure Gas Thermometer.*—Taking τ_0 as the temperature of melting ice and $\tau_0 + 100$ as the temperature of boiling water—*i. e.*, taking 100 degree-divisions of the τ-scale of temperature between these two fixed points—it is found (Gay-Lussac obtained the value 0.391, later corrected to 0.366) that under atmospheric pressure

$$\frac{v(p, \tau_0 + 100) - v(p, \tau_0)}{v(p, \tau_0)} = 0.366.$$

If, now, any temperature τ on this scale be considered such that $\tau - \tau_0$ is proportional to the volume increase relative to the volume at the temperature of melting ice, then

$$k(\tau - \tau_0) = \frac{v - v_0}{v_0} \text{ at constant pressure.}$$

When

$$\tau = \tau_0 + 100, \quad k = 0.00366.$$

The relation is further simplified by writing $\tau_0 = 273$; then

$$\tau = \frac{273}{v_0} v.$$

A similar definition of the temperature scale of a constant-volume gas thermometer could be stated:

$$\tau = \frac{273}{p_0} p.$$

The two equations for the τ-temperature of a body are also formulations of the two laws of Gay-Lussac, and as such will be employed hereafter in place of the more cumbersone forms first stated.

Since the quantities v_0 and p_0 are functions of p and v respectively, the two laws of Gay-Lussac can be written

$$\frac{v}{\tau} = \psi_1(p), \tag{2a}$$

$$\frac{p}{\tau} = \psi_2(v). \tag{2b}$$

THE FREE-EXPANSION EFFECT.

(*a*) *Zero Free-expansion Effect.*—When a mass of one of the "permanent" gases expands adiabatically into a vacuum, the temperature of the mass remains practically unchanged. This was first observed by Gay-Lussac, and again much later by Joule. Assuming the temperature to remain constant,

$$\tau(e, v_2) - \tau(e, v_1) = 0,$$

where e is the specific energy of the gas; and

$$e(v_2, \tau) - e(v_1, \tau) = 0;$$

whence it appears that both τ and e are independent of v, *i. e.*,

$$\left(\frac{\partial \tau}{\partial v}\right)_e = 0, \tag{3}$$

and

$$\left(\frac{\partial e}{\partial v}\right)_\tau = 0.$$

(*b*) *Formulation of the Free-expansion Effect.*—In order to obtain general expressions for these two derivatives, the procedure is as follows: If in the differential equation for the energy expressed as a function of v and s

$$de = -pdv + \theta ds,$$

where s is the specific entropy of the body, and $\theta(\tau)$ is the thermodynamic temperature, the variables v, τ be taken independent, the equation becomes

$$de = \left(-p + \theta \frac{\partial s}{\partial v}\right) dv + \theta \frac{\partial s}{\partial \tau} d\tau,$$

and the condition for an exact differential yields

$$-\frac{\partial p}{\partial \tau} + \frac{d\theta}{d\tau}\frac{\partial s}{\partial v} = 0;$$

whence

$$\left(\frac{\partial \tau}{\partial v}\right)_e = \frac{-\left(-p + \theta \frac{\partial p}{\partial \tau}\frac{d\tau}{d\theta}\right)}{c_v(v, \tau)}, \tag{4}$$

since $\theta(\partial s/\partial \tau) = c_v(v, \tau)$, the specific heat at constant volume per degree of temperature τ. Also

$$\left(\frac{\partial e}{\partial v}\right)_\tau = \left(-p + \theta \frac{\partial p}{\partial \tau}\frac{d\tau}{d\theta}\right).$$

(*c*) *The Relation between the Thermodynamic Temperature and the Free-expansion Effect.*—Solving equation (4) for θ:

$$\frac{d \log \theta}{d\tau} = \frac{\dfrac{\partial p}{\partial \tau}}{p - c_v(v, \tau)\left(\dfrac{\partial \tau}{\partial v}\right)_e}.$$

When the quantities in the second member of this equation are experimentally determined with sufficient accuracy, as functions of the variables v. τ, the relation between the arbitrary scale of temperature and the thermodynamic scale is established.

(*d*) *Consequence of the Law of Zero Free-expansion Effect.*—From the above equation, writing

$$\left(\frac{\partial \tau}{\partial v}\right)_e = 0$$

it follows, since $c_v(v, \tau)$ is always finite, that

$$\frac{d \log \theta}{d\tau} = \frac{\partial \log p}{\partial \tau},$$

whence on integrating

$$\frac{p}{\theta} = f_1(v). \tag{5}$$

This is an equivalent expression of the law of zero free-expansion effect.

The Porous-Plug Effect.

(*a*) *Zero Porous-plug Effect.*—The measurement of the free-expansion effect necessitates the use of bulky apparatus, and, since the effect is at best very small, its measurement is extremely uncertain. In order to determine with greater accuracy the temperature change in an expansion, Joule and Thomson devised the so-called porous-plug experiment. In this operation a steady state is reached during which gas, under a constant initial pressure, is forced through a porous plug, and escapes under a constant final pressure—the whole apparatus being thermally insulated.

Here the work pv is added to unit mass of the gas in the initial stage of the operation, while the work $(p + \delta p)(v + \delta v)$ is developed by the gas in the concluding stage. The process being adiabatic, the change of energy is

$$\delta e = -\delta(pv),$$

that is,

$$\delta(e + pv) = 0, \quad e + pv = \text{const.}$$

This is the condition under which the experiment is made; and the derivative

$$\left(\frac{\partial \tau}{\partial p}\right)_{e+pv}$$

is called the porous-plug effect. Under different conditions of pressure and temperature this may be positive, zero, or negative. It is of interest to determine the consequences of assumption of a zero porous-plug effect, *i. e.*,

$$\left(\frac{\partial \tau}{\partial p}\right)_{e+pv} = 0. \tag{6}$$

An equivalent expression is

$$\left(\frac{\partial (e + pv)}{\partial p}\right)_{\tau} = 0.$$

(*b*) *Formulation of the Porous-plug Effect.*—From the differential equation for the specific energy, we obtain, on adding $d(pv)$ to both members,

$$d(e + pv) = vdp + \theta ds.^{1}$$

Taking p, τ as independent variables,

$$d(e + pv) = \left(v + \theta \frac{\partial s}{\partial p}\right) dp + \theta \frac{\partial s}{\partial \tau} d\tau;$$

and the condition for an exact differential yields

$$-\frac{\partial s}{\partial p}\frac{d\theta}{d\tau} = \frac{\partial v}{\partial \tau};$$

whence

$$\left(\frac{\partial \tau}{\partial p}\right)_{e+pv} = \frac{-\left(v - \theta \frac{\partial v}{\partial \tau}\frac{d\tau}{d\theta}\right)}{c_p(p, \tau)}; \tag{7}$$

since $\theta(\partial s/\partial \tau) = c_p(p, \tau)$, which is the specific heat at constant pressure per degree of temperature τ.

(*c*) *Relation between the Thermodynamic Temperature and the Porous-plug Effect.*—Solving equation (7) for θ:

$$\frac{d \log \theta}{d\tau} = \frac{\frac{\partial v}{\partial \tau}}{v + c_p(p, \tau)\left(\frac{\partial \tau}{\partial p}\right)_{e+pv}}.$$

[1] This fundamental function $(e + pv) = g(p, s)$ is the χ-function of Gibbs, called by him the heat function at constant pressure. Kammerlingh-Onnes has suggested for it the name "enthalpy."

When the quantities in the second member of this equation are experimentally determined with sufficient accuracy, as functions of the variables p, τ, the relation between the arbitrary scale of temperatures and the thermodynamic scale is established.

(*d*) *Consequence of the Law of Zero Porous-plug Effect.*—From the above equation, writing

$$\left(\frac{\partial \tau}{\partial p}\right)_{e+pv} = 0,$$

it follows, since $c_p(p, \tau)$[1] is always finite, that

$$\frac{d \log \theta}{d\tau} = \frac{\partial \log v}{\partial \tau};$$

whence on integrating

$$\frac{v}{\theta} = f_2(p). \tag{8}$$

This is an equivalent expression of the law of zero porous-plug effect.

The Characteristic Equation of an Ideal Gas.

It is now possible to deduce consequences of the laws of Boyle, of Gay-Lussac, of zero free-expansion effect, and of zero porous-plug effect, taken two at a time.

(*a*) The laws of Boyle and zero free-expansion effect lead at once to the relation $pv = r\theta$; for, eliminating p between equations (1) and (5),

$$pv = F[\tau(\theta)], \quad \frac{p}{\theta} = f_1(v),$$

and separating the variables,

$$\frac{F[\tau(\theta)]}{\theta} = f_1(v) \cdot v = r;$$

whence

$$pv = r\theta.$$

This is the so-called "characteristic equation" of an ideal gas. It is a relation between the pressure, specific volume, and thermodynamic temperature of a one-component, one-phase body, approximately valid in a limited region of vapor states of the body.

(*b*) The laws of Boyle and of zero porous-plug effect in similar manner lead at once to the relation $pv = r\theta$; as is seen by inspecting equations (1) and (8),

$$pv = F[\tau(\theta)], \quad \frac{v}{\theta} = f_2(p).$$

[1] Here c_p denotes the specific heat of an ideal gas.

(*c*) The first law of Gay-Lussac and the law of zero free-expansion effect, when combined, lead to no simple expression.

The second law of Gay-Lussac and the law of zero free-expansion effect require the temperatures of the thermodynamic scale and those of the constant-volume gas thermometer to be proportional for all volumes. See equations in section *e*.

(*d*) The first law of Gay-Lussac and the law of zero porous-plug effect require the temperatures of the thermodynamic scale and those of the constant-pressure gas thermometer to be proportional for all pressures. See equations in section *e*.

The second law of Gay-Lussac and the law of zero porous-plug effect when combined lead to no simple expression.

(*e*) Just as the laws of Boyle and the two laws of Gay-Lussac are interdependent, so are the laws of Boyle, of zero porous-plug effect, and of zero free-expansion effect; as is evident by inspection of the following equations:

$$(1)\quad pv = F(\tau), \qquad (1)\quad pv = F[\tau(\theta)],$$

$$(2a)\quad \frac{v}{\tau} = \psi_1(p), \qquad (8)\quad \frac{v}{\theta} = f_1(p),$$

$$(2b)\quad \frac{p}{\tau} = \psi_2(v), \qquad (5)\quad \frac{p}{\theta} = f_2(v).$$

Specific Heats.

It will be shown later, that in addition to the characteristic equation, some knowledge of specific heats is necessary to define a one-component body in a given region of states.

(*a*) *The Difference of the Specific Heats.*—Evaluation of the general relation

$$c_p - c_v = \theta \left(\frac{\partial p}{\partial \theta}\right)_v \left(\frac{\partial v}{\partial \theta}\right)_p$$

by means of the characteristic equation $pv = r\theta$, leads at once to the result

$$c_p - c_v = r.$$

(*b*) *Regnault's Law and its Consequences.*—It was observed by Regnault that, for the "permanent" gases, the specific heat at constant pressure is practically constant, *i. e.*, independent of either pressure or temperature,

$$c_p(p, \theta) = \text{const.}$$

Thence follows that

$$\frac{\partial c_p}{\partial p} = 0 \quad \text{and} \quad \frac{\partial c_p}{\partial \theta} = 0.$$

For want of better names, these formulations will be called statements of Regnault's first and second laws, respectively.

The first law permits partial determination of the characteristic equation; for, since in general

$$\frac{\partial c_p}{\partial p} = -\theta \frac{\partial^2 v}{\partial \theta^2},$$

it follows on integration that

$$\frac{\partial v}{\partial \theta} = \varphi_1(p), \tag{9a}$$

which is the differential form of the consequence of the law of zero porous-plug effect.

In an analogous manner, if one expresses a law of constancy of the specific heat at constant volume, it follows that

$$\frac{\partial c_v}{\partial v} = 0 \quad \text{and} \quad \frac{\partial c_v}{\partial \theta} = 0,$$

and from the first equation

$$\frac{\partial c_v}{\partial v} = 0 = \theta \frac{\partial^2 p}{\partial \theta^2};$$

whence

$$\frac{\partial p}{\partial \theta} = \varphi_2(v), \tag{9b}$$

which is the differential form of the consequence of the law of zero free-expansion effect.

Moreover the law of Boyle and the consequences of the two laws $\partial c_p/\partial p = 0$ and $\partial c_v/\partial v = 0$ show the same interdependence as do the law of Boyle and the two laws of Gay-Lussac; or the law of Boyle, the law of zero free-expansion effect, and the law of zero porous-plug effect—as is evident on comparison of the groups of equations: (1), (2*a*), (2*b*); (1), (8), (5); (1), (9*a*), (9*b*).

On the other hand the equation

$$\frac{\partial c_p}{\partial \theta} = 0$$

is in no manner connected with the characteristic equation. The same statement applies to the equation

$$\frac{\partial c_v}{\partial \theta} = 0.$$

It is obvious, therefore, that either of these equations affords knowledge of the thermodynamic properties of a body, not to be obtained from the characteristic equation. Yet, since $(c_p - c_v)$ depends solely upon the form of the characteristic equation, the temperature derivative of either can be found from that of the other.

FUNDAMENTAL FUNCTIONS OF A ONE-COMPONENT, ONE-PHASE BODY.

Gibbs[1] has shown that for any one-component body in any region of one-phase states, there exist three independent relations between the six variables p, θ, h, V, S, M (the pressure, temperature, potential, volume, entropy and mass of the body). Any definition from which three such independent relations can be deduced is a complete definition of the body. Further, he has shown that a relation between e, v, s; f, v, θ; g, p, s; or h, p, θ constitutes such a definition (where f is the specific free-energy of the body).

(*a*) *Sufficient Relations for the Formulation of the Fundamental Functions.*—To illustrate the above statement, consider, for example, the specific free-energy of the body. Then

$$df(v, \theta) = \frac{\partial f}{\partial v} dv + \frac{\partial f}{\partial \theta} d\theta = - pdv - sd\theta,$$

and

$$ds(v, \theta) = \frac{\partial s}{\partial v} dv + \frac{\partial s}{\partial \theta} d\theta = \frac{\partial p}{d\theta} dv + \frac{c_v}{\theta} d\theta,$$

and also

$$dc_v(v, \theta) = \frac{\partial c_v}{\partial v} dv + \frac{\partial c_v}{\partial \theta} d\theta = \theta \frac{\partial^2 p}{\partial \theta^2} dv + \frac{\partial c_v}{\partial \theta} d\theta.$$

Consequently, if the characteristic equation together with the form of $\partial c_v(v, \theta)/\partial \theta$ is known, one has sufficient data with which to obtain the specific free energy as a function of v, θ; and hence to formulate the thermodynamic properties of the body in that region of states for which the characteristic equation and the expression for the specific heat derivative are valid.

In similar manner, the specific thermodynamic potential can be determined from the characteristic equation and the form of $\partial c_p(p, \theta)/\partial \theta$.

[1] Gibbs, Scientific Papers, I., p. 88.

(*b*) *The Definition of an Ideal Gas.*—From the above two examples, it is seen that the form of the characteristic equation and the form of either $\partial c_v/\partial\theta$ or $\partial c_p/\partial\theta$ are sufficient to define a one-component body thermodynamically in a region of its one-phase states. In the particular case of the ideal gas there is a common consensus of opinion that the characteristic equation $pv = r\theta$ is an essential feature of its definition, but the further necessity of the second law of Regnault—that the temperature derivative of one of the specific heats is zero—is generally neglected.

As an illustration, consider the behavior of two gases on reversible adiabatic expansion, both gases having the characteristic equation $pv = r\theta$, while the specific heat temperature derivative of the first is zero, and that of the second is constant and finite,

$$\text{I. } \frac{\partial c_v}{\partial\theta} = 0, \quad \text{II. } \frac{\partial c_v}{\partial\theta} = b_2.$$

Then for both:

$$ds = \frac{\partial s}{\partial v}dv + \frac{ds}{\partial\theta}d\theta$$

$$= \frac{\partial p}{\partial\theta}dv + \frac{c_v}{\theta}d\theta,$$

and for each, since

$$\text{I. } c_v = a_1, \qquad \text{II. } c_v = a_2 + b_2\theta;$$

therefore

$$ds = \frac{r}{v}dv + \frac{a_1}{\theta}d\theta, \quad ds = \frac{r}{v}dv + \frac{a_2 + b_2\theta}{\theta}d\theta,$$

and

$$\left(\frac{\partial v}{\partial\theta}\right)_s = -\frac{va_1}{r\theta}, \qquad \left(\frac{\partial v}{\partial\theta}\right)_s = -\frac{v}{r\theta}(a_2 + b_2\theta).$$

These differing results indicate the insufficiency of the equation $pv = r\theta$ as a definition of an ideal gas.

Summary.

The preceding is an attempt at logical development of the concept of an ideal gas. After the many discussions of the subject in the past, it is needless to state that no original conclusions are reached; it is hoped merely that the method of presentation has some advantages. It is a pleasure to acknowledge my indebtedness to Professor Trevor for his criticism and advice in the preparation of this paper.

Cornell University,
January, 1912.

A RELATION BETWEEN THE MAGNETIC AND ELASTIC PROPERTIES OF A SERIES OF UNHARDENED IRON-CARBON ALLOYS.

BY C. W. WAGGONER.

THIS paper is a continuation of a study of some of the physical properties of a series of annealed iron-carbon alloys; the magnetic hysteresis and coefficient of linear expansion having been presented in a previous paper.[1] For convenience the chemical analysis and heat treatment have been repeated here in Table I. below.

TABLE I.

Chemical Analysis.

Mark.	C	P	Si	Mn	S
P.I.	.058	Trace	.008	.071	—
A1	.60	.013	.15	.14	.012
A2	.74	.012	.16	.14	.013
A3	.89	.010	.19	.155	.013
A4	.98	.012	.16	.15	.013
A5	1.18	.012	.14	.14	.013
A55	1.26	.012	.16	.17	.014
A6	1.37	.011	.19	.16	.012

Heat Treatment.—The steels were all placed in a platinum resistance furnace and heated to 1000° C. After keeping them at that temperature for two hours, they were allowed to cool slowly in the furnace.

THE MAGNETIC HYSTERESIS AT 20° C.

In the previous investigation it was found that the hysteresis loss, at ordinary temperatures, increases with the carbon content up to about 1.1 per cent. of carbon and then decreases with increasing carbon. The limited series of steels, and the fact that the results did not indicate clearly the maximum of the curve of hysteresis loss and the percentage of carbon, led the writer to plot some data given by Benedicks[2] for a series of annealed steels. Benedicks' data when plotted showed a decided

[1] Waggoner, PHYS. REV., Vol. XXVIII., p. 393, 1909.

[2] Benedicks, Thèse Pour Le Doctorat, Uppsala, 1904.

maximum and the writer felt justified in drawing his curve in a like manner. These curves are shown in Fig. 1 as taken from a previous paper. The fact that the maximum of the hysteresis-carbon curve should lie so near the eutectic point of the iron-carbon series, was pointed out at the time, but the writer did not feel that the data were sufficient to justify any speculation as to the significance of this fact. Further study has, however, led to the conclusion that the shape of the hysteresis-carbon curve may be explained by a consideration of the physical properties of the microscopic constituents of the iron-carbon series.

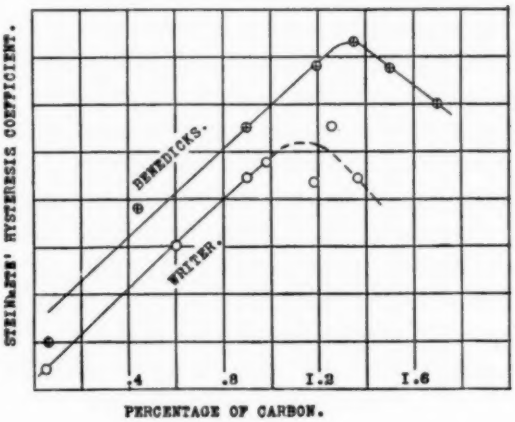

Fig. 1.

Showing the relation between the hysteresis loss and carbon content.

Sauver[1] calculated the proportions of the three constituents present in unhardened steels from an analysis of cementite and pearlite. A table showing the results of more recent investigation is however, given by Howe[2] and is reproduced in Fig. 2. An examination of the constituents of unhardened steels as given in Fig. 2 shows pure iron to be made up of a microscopic constituent called Ferrite. This constituent, Howe gives[3] as the microscopic particles of nearly pure iron. It is magnetic, soft and ductile, having a tensile strength of about 45,000 pounds per square inch.

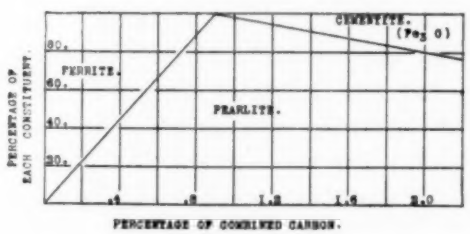

Fig. 2.

Showing the constitution of iron-carbon compounds.

Carbon dissolves in iron, forming a definite carbide of iron, Fe_3C, called cementite. This microscopic constituent is extremely brittle, harder than hardened steel, and magnetic (but probably much less so than ferrite).

[1] Sauver, Am. Inst. of Min. Eng., Sept., 1896.
[2] Howe, Iron, Steel and other Alloys, p. 184.
[3] Loc. cit., p. 175.

In a steel containing about 1 per cent. carbon (the eutectic) ferrite and cementite form in very closely interstratified layers, representing perhaps the most intimate mixture of these two constituents possible. This combination of ferrite and cementite, because of its appearance under the microscope, has been called pearlite. From the figure it will be seen that a low carbon steel consists of a mixture of this pearlite and an excess of pure iron or ferrite. As the eutectic percentage of carbon and iron is approached the amount of free ferrite grows less, and at 1 per cent., or near it, the alloy consists entirely of this closely-woven mixture of ferrite and cementite. Beyond the eutectic percentage the alloy consists of pearlite and an excess of cementite.

Assuming now, that the hysteresis loss in an alloy is due to the magnetic reversal of polarity of the iron molecules when placed in an alternating field, it seems probable that the increase in the hysteresis loss up to the eutectic may be due to the decrease in free ferrite, and the increase in pearlite; the opposition to the reversal of polarity increasing with the decreasing percentage of free ferrite. It seems reasonable, also, to suppose that the magnetic molecules of cementite and the molecules of ferrite, bound up so closely to form pearlite, would not possess the freedom of molecular vibration under the alternating magnetic field as would free, uncombined ferrite molecules. Following further this method of reasoning, the hysteresis loss should be a maximum at the eutectic percentage, since at this point we have ferrite and cementite forming a mixture so intimate that the separate layers may only be detected microscopically when very high magnifications are used.

That the hysteresis loss decreases, when the carbon content is in excess of the eutectic percentage, may be due to the freeing of the ferrite molecules in the solid mass of pearlite by the net work of excess cementite which surrounds the pearlite crystals and separates them from each other, making it easier for the ferrite molecules to follow the alternations of the impressed magnetic field. Of course this decrease in hysteresis loss does not continue with increasing cementite but must reach a minimum, where the advantage gained by loosing the bound molecules of ferrite is met by the decreasing number of them present in the alloy.

INTENSITY OF MAGNETIZATION.

From the reasoning given above for the hysteresis-carbon curve it is clear that the intensity of magnetization, at any given magnetic field strength high enough to produce saturation in the steel, should decrease with the carbon content. The increasing percentage of pearlite should result in a smaller angular displacement of the magnetic molecule under

the influence of the magnetic field, thus decreasing the magnetic intensity. A further decrease in the intensity should be produced by reason of the smaller number of free ferrite molecules as the pearlite approaches the eutectic percentage. The intensity of magnetization should increase with the percentage of carbon in excess of the eutectic on account of the breaking up of the solid pearlite by the excess cementite, thus freeing some of the ferrite molecules and allowing them larger angular displacement in the magnetic field.

To examine this point the writer has made use of some data taken by Dorsey,[1] to whom these alloys were loaned for a study of the magnetostriction. In the magnetostriction experiments the intensity of magneti-

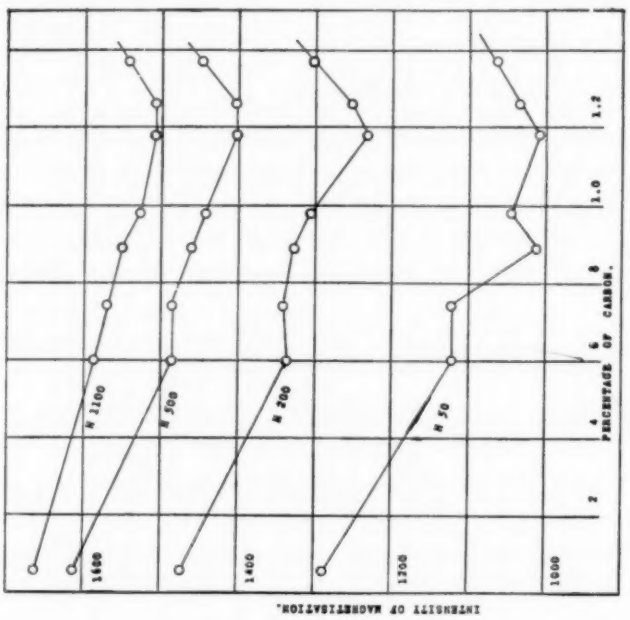

Fig. 3.

Showing the relation between the intensity of magnetization at various field strengths and the carbon content.

zation was determined ballistically, proper corrections being made for the short length of the test bars. By interpolation from the data given, the intensity of magnetization was found for each alloy at the following fields: 50, 200, 500 and 1,100 C.G.S. Fig. 3 shows the curves of intensity of magnetization when plotted against the carbon content.

It will be seen that the minimum of the curves in Fig. 3 corresponds very well with the maximum of the curve in Fig. 1, *i. e.*, near the eutectic

[1] Dorsey, PHYS. REV., Vol. XXX., p. 698, 1910.

percentage. The curves in Fig. 3 further show that the intensity of magnetization at high fields is affected very much less by increasing carbon content than at low fields. In other words, as complete magnetic saturation is approached, the intensity of magnetization is less affected by the chemical composition of the alloy.

The results shown in Fig. 3 are not in agreement with Benedicks,[1] who finds that at a magnetization of 205 C.G.S., the intensity of magnetization decreases only very slightly with the increase in carbon content. There is no indication in his results of a minimum in the curve of intensity of magnetization and carbon content at the eutectic percentage for the steels he studied, but the decrease in intensity of magnetization with the carbon content is very much more marked when the carbon exceeds 1.3 per cent.

The Elastic Properties and the Carbon Content.

The relation between the maximum strength of steels and the carbon content has been the subject of a large number of investigations, but in many cases the results are useless because the heat treatment has not been uniform or because of large variation in the constituents other than the carbon. In order to make this investigation complete a series of

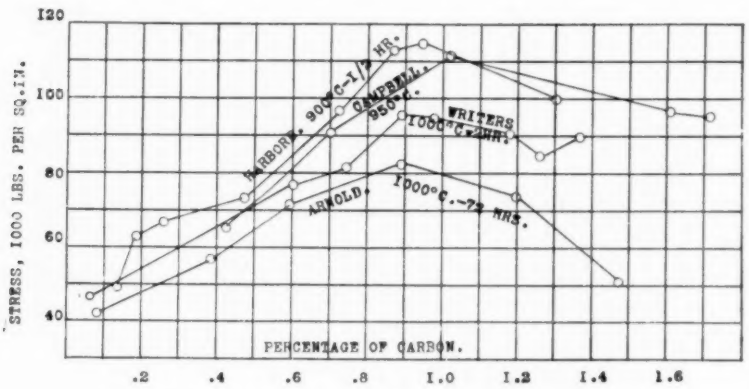

Fig. 4.

Showing the relation between the maximum strength and the carbon content.

strength tests were made on a 20,000 pound Riehle Standard Testing Machine and the complete results were made the subject of another paper.[2] For purposes of comparison with the magnetic properties, the writer has taken the liberty of reproducing here in Fig. 4, a diagram showing the relation between the maximum strength and the carbon

[1] Benedicks, loc. cit., p. 160.

[2] Jones and Waggoner, Proc. A. S. for Testing Mat., Vol. XI., 1911.

content. This figure includes, beside the series of alloys under investigation, some other published results.

An inspection of the shape of the curves in Fig. 4 shows a very striking similarity to the curves for the hysteresis loss given in Fig. 1, with the maximum strength at, or near, 1 per cent. of carbon. The difference between the several maximum strength curves is not greater than could be reasonably expected from the difference in the composition and heat treatment. The curves fall in the order in which they would be logically placed by the methods of annealing employed. The particular heat treatment given the steels under test was not chosen to bring out the highest tensile strength, but because these steels had a similar heat treatment for the magnetic tests.

The shape of the maximum strength-carbon curves had been predicted by Howe[1] from the physical characteristics of the microscopic constituents. Ferrite, being soft and ductile has a relatively low strength. As the carbon content increases the ferrite is surrounded by a network of pearlite, thus increasing the adhesion between the crystals. That the strength of unhardened steels reaches a maximum at or near the eutectic is largely due to the small crystallization and very intimate mixture of the crystalline constituents. On the other hand when the eutectic is passed the pearlite is surrounded with a network of cementite and both of these networks have a tendency to destroy the adhesion between the crystals.

It would seem then that the magnetic hysteresis and the maximum strength of steels vary in the same way with changing carbon content; that the similarity between the magnetic and elastic hysteresis is more than a mere similarity of shape of their characteristic curves. That the magnetic and elastic properties are related in some fashion has been known for a long time. Ryder[2] proposed to classify steels by the residual magnetism of short bars, and Osmond[3] in commenting on this said, that in studying this method of classification he had found it in practical accordance with that based on tensile strength. "A great advantage of these magnetic tests," he said, "is that they could be applied to the finished pieces, and not merely to a test-piece more or less restricted as to shape."

A further similarity is shown[4] in the fact that increasing the density of steel by cooling it in liquid air increases the hysteresis loss when the steel

[1] Howe, loc. cit., p. 162.
[2] Ryder, Eng. and Min. Jour., 1877.
[3] Osmond, Pro. I. C. E., Vol. CXXVI., p. 264.
[4] Waggoner, loc. cit.

has been magnetized to high inductions, over the loss of hysteresis at ordinary room temperatures; and Hadfield[1] has shown that the low temperature also increases the tensile strength. Hardening steels by other methods results in an increase in both properties; while proper annealing will reduce both factors in the same way.

A comparison of the percentage elongation under stress, or the ductility, when plotted against the carbon content, with the intensity of magnetization when plotted in a like manner, shows that the curves are strikingly similar, indicating that the ductility of these alloys and their intensity of magnetization are affected in the same way by the chemical composition. The maximum susceptibility-carbon curve is also similar to the curve of ductility-carbon, *i. e.*, the maximum susceptibility decreases with increasing carbon until the eutectic is reached and then again increases with the increase in carbon content.

The Magnetostriction and the Carbon Content.

Dorsey, in his work on the magnetostriction in these alloys, has shown that the maximum percentage elongation of the bar when plotted against the percentage of carbon is exactly similar to the maximum susceptibility curve. Assuming now that magnetostriction is nothing more than an attempt on the part of the ferrite molecules to orient themselves in a given way under the influence of the magnetic field, the shape of the magnetostriction-carbon curve with its minimum at, or near, the eutectoid, may be explained in a manner similar to that given above for the maximum strength and hysteresis; *i. e.*, it would seem reasonable to expect the smallest percentage elongation in a steel having the eutectic percentage of pearlite, since this percentage of ferrite and cementite is such as to produce the maximum adhesion between the crystals.

Summary.

1. In a series of unhardened steels, the relation between the hysteresis loss, intensity of magnetization, maximum susceptibility and the carbon content might have been predicted from the Ewing theory of magnetism and a knowledge of the physical characteristics of the microscopic constituents of such steels.

2. The magnetic hysteresis loss and the maximum strength of unhardened steels vary in the same way with the percentage of carbon.

3. The intensity of magnetization, at saturation fields, and the maxi-

[1] Hadfield, J. I. and S. Inst., Vol. LXVII., p. 143, 1905.

mum susceptibility show a like variation with carbon content, as the ductility of these steels.

4. The maximum percentage elongation due to magnetostriction in a series of annealed steels seems to be a function of the ductility of the steel and may be explained on a basis of the elastic properties of the microscopic constituents comprising these steels.

PHYSICAL LABORATORY,
WEST VIRGINIA UNIVERSITY,
March 28, 1911.

THE CRYSTALLIZATION OF CARBON-DIOXIDE, NITROUS OXIDE AND AMMONIA.

BY H. E. BEHNKEN.

A DISCUSSION by Professor Gill, of the similarity of the chemical compounds of carbon and silicon, during a study of the quartz crystal, suggested the problem of determining the crystallization of the dioxide of carbon.

Several methods for securing crystals were considered and tested.

1. The solution of the gas in a liquid of low freezing point and a microscopic study of the crystals formed on reducing the temperature below that of the sublimation of CO_2 crystals. This was tried with petroleum ether as the liquid, but no satisfactory study of the resulting cloudiness was made.

2. The freezing of the liquified gas.

3. The solution of the gas in some cold liquid of low boiling point and the production of crystals on evaporation.

4. The production of crystals by direct freezing. Two schemes were proposed; either to pass the gas through a very cold liquid, or to allow it to come in contact with a cold solid surface. The latter proved successful.

The chief difficulty was met in the elimination of water vapor, the crystals of which might not be distinguishable or whose presence in quantity would make results uncertain. While this would appear a simple matter, with the powerful dehydrating reagents available, it was not found possible to prevent the formation of some ice crystals.

The essentials of the apparatus, as suggested by Professor Shearer, consisted of a metal cup into which the gas was directed, a microscope for studying the crystals, and a Dewar cylinder of liquid air to be used as the freezing agent; the cooling was controlled by means of a copper rod attached to the bottom of the cup and immersed in the liquid air. By varying the distance between the surface of the cold liquid and the cup, the temperature of the freezing space could be regulated.

Difficulty in illuminating the field on an opaque surface in a Dewar tube caused the substitution of a glass freezing plate. By means of a series of reflections, the light could be sent up through the plate.

An arc light (*A*), Fig. 1, was used as the source of illumination, the heating effect of which was avoided by passing the light through the water cell (*W*). The light was then converged (*C*) and reflected down into the Dewar by the mirror (*M*).

The light in the Dewar (*D*) was reflected at right angles by a prism (*P*) to a small concave mirror (*R*) set in the freezing tube (*T*). This mirror was not actually made, being in fact a piece cut out of a broken silvered Dewar bulb. It was set at approximately 45° as shown, being held in place by pieces of tubing, cut at an angle, and slipped inside the main tube. After this reflection, the light passed through the freezing stage (*S*) to the microscope.

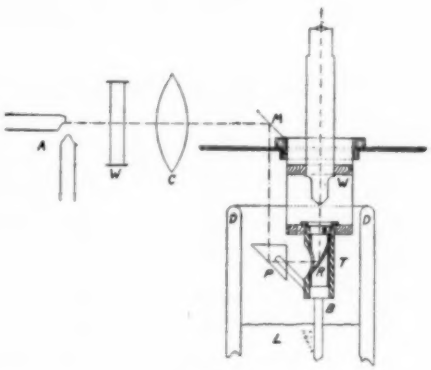

Fig. 1.

A, arc; *W*, water; *C*, condensing lens; *M*, mirror; *DD*, Dewar cylinder; *P*, reflecting prism; *L*, liquid air; *T*, freezing tube; *R*, concave mirror; *S*, glass freezing stage; *W*, wooden disk piston on microscope.

What has been spoken of as the "freezing tube" is a short length of Tobin bronze tubing, about 7/8 inch outside and 5/8 inch inside measure. A rabbet was cut around the top to hold the piece of glass on which the crystals were to form; this was a microscope cover glass, cut down to fit. To this tube was connected a brass rod (*B*) which dipped into the liquid air (*L*) and by means of which the temperature could be controlled. A hole was bored in the side of the freezing tube to admit the light to the mirror inside.

To exclude the outside air with its attendant moisture, the freezing plate was enclosed in a chamber made of a four-inch length of 2½ inch thin glass tubing. The bottom of this chamber consisted of a wooden disk, cemented in with shellac, and thoroughly coated with the same. Through this disk, a hole was bored which admitted the freezing tube, a collar soldered to it holding it up. This hole being eccentric, allowed a limited horizontal shifting of the field under the microscope by turning the chamber. To allow of the focusing of the microscope, it was equipped with a fitted wooden disk covered with vaseline (*W*) which, by acting as a piston in the chamber, kept out the air completely. The CO_2 was admitted by a glass tube passing through a hole cut in the glass wall of the freezing chamber; all the space in this hole not occupied by the tube was filled with a lump of vaseline. To aid in keeping this chamber dry, a small boat filled with P_2O_5 was introduced.

Some difficulty was encountered in keeping the reflecting prism and other surfaces free of frost. This was partially overcome by enclosing the entire system so that the evaporated liquid air, which was practically dry, filled the enclosure. Even with this, frost slowly gathered and the prism must be warmed at intervals.

To allow the raising and lowering of the Dewar without admitting outside air, a square telescope box was used. The entire arrangement

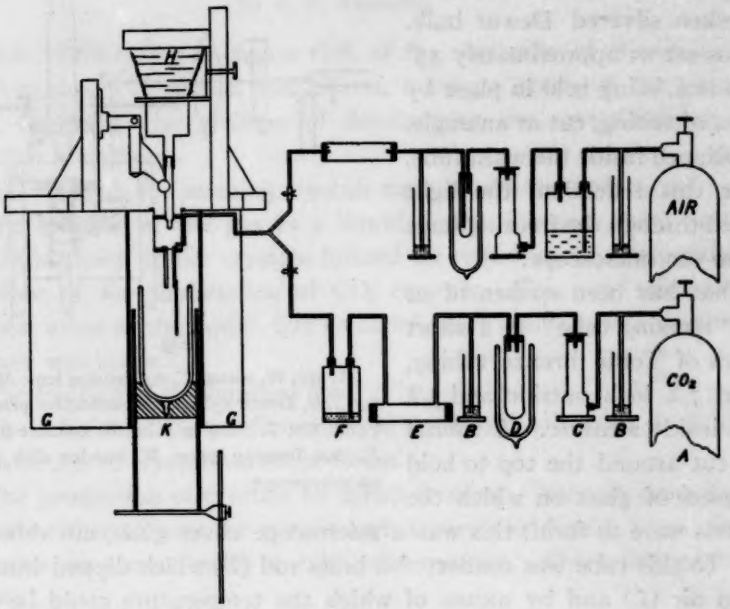

Fig. 2.

(*A*) Tank of liquid CO_2. (*B*) Safety by-passes, T tubes with end dipping below surface of mercury in cylinder. The depth of immersion determines safety point. (*C*) Calcium chloride drying tower. (*D*) Test-tube with Dewar tube of liquid air in which it can be immersed to various depths. Dewar tube is on adjustable stand. (*E*) Drying tube filled with glass wool dusted with P_2O_5. (*F*) Bottle with small amount of mercury by means of which the rate of flow of the gas could be watched. (*G*) Large glass case serving as enclosure and as support for bulk of apparatus. (*H*) Camera on adjustable stand. (*K*) Telescoping box holding large Dewar cylinder; supported by stand under table which rests on floor. Inner box of telescoping box passes through hole in table.

was enclosed in a large glass case, which, beside serving as an enclosure, supported the microscope, camera, freezing chamber, and reflector as shown in Fig. 2. The outer, fixed tube of the telescoping box extended from the top of the table to the top of this case, and was equipped with a strip of glass through which to observe the height of the Dewar. The inner, sliding tube, which was closed by the base carrying the Dewar,

passed through a hole cut in the table to fit it, but allow it to slide up and down. A quantity of $CaCl_2$ in the outer case helped to keep the moisture down. The tube carrying the Dewar was supported by an adjustable iron stand resting on the floor.

Liquid CO_2 in cylinders was allowed to vaporize and the gas was purified by freezing it in a tube immersed in liquid air, on allowing the solid to sublime slowly, the rate being controlled by the depth of immersion, a high degree of purity was secured.

The greatest problem was the elimination of moisture from the gas. This has not been entirely mastered, for ice made its appearance in spite of all precautions. The CO_2 from the tank was first passed slowly through a $CaCl_2$ drying tower 12 inches high. It was then frozen in liquid air as described above. Before the gas secured by sublimation was admitted to the freezing chamber, it was passed through a 14-inch condenser tube filled with glass wool well filled with P_2O_5. As an additional precaution a small boat of P_2O_5 was placed in the freezing chamber. But in spite of all precautions, fairly large ice crystals were formed after passing the gas into the chamber for a half hour at a temperature too high to form CO_2 crystals. The fact that long crystals (needles) of this ice grew on top of the layer of phosphoric acid formed over the P_2O_5 in the boat showed the inefficiency of this agent under these circumstances, or when long runs are made. As the temperature rose, the unchanged P_2O_5 absorbed this moisture. Of course this did not occur in the short runs necessary for the production of the CO_2 crystals.

To test the efficiency of the drying arrangement and to make sure that CO_2 was not necessary to the production of this form, a duplicate freezing and drying train was built, through which air purified by lime, solid KOH, and KOH solution was passed. This pure air produced the same crystals, indicating that moisture alone was the source. This train was connected to the other by a Y tube and was useful in flushing the freezing chamber and also, by supplying air during cooling, preventing the drawing in of atmospheric air by contraction of the gas in the chamber.

In forming the crystals, different procedure was found to give great difference in results. If a considerable stream of CO_2 was allowed to flow into the chamber before and during the cooling, the plate would be covered rapidly, when the right temperature was reached, by crystals in fern leaf forms. These would quickly unite and produce a solid covering. If the plate were barely cold enough, they would begin to sublime on shutting off the CO_2. If the plate was first well cooled, and the stream was turned directly on it, the same forms were produced.

The best results were obtained by delivering the gas tangentially at the edge of the cooling stage. In this way, the gas would be delivered on the plate gradually, although it might come out of the tube at a greater rate. Much of it was frozen on the brass, sometimes producing a transparent glaze. This procedure caused the gas reaching the glass plate to be gradually cooled, maintaining more even conditions and tending to better crystal formation.

The best crystals were obtained by reducing the plate to a moderately cold condition and then admitting the gas in *limited* quantities. A quantity of solid CO_2 was first collected, the non-freezing portions being allowed to escape to the air by one of the safety vents. The solid CO_2 was then allowed to sublime slowly, flushing out the apparatus. The CO_2 was then shut off and the pure air turned into the chamber, sweeping out all residual gas. The Dewar cylinder having been filled with liquid air by means of a tube passing through a hole in the box, was then raised to bring the level of the liquid air about three inches below the freezing plate. The passing of the dry air prevented the drawing in of the outer air on cooling. The air is now shut off, the CO_2 being turned on at the same time. Only a small amount must be admitted at this time, as otherwise too many crystals are formed. They seem to flash into existence. By feeding the CO_2 very slowly or in a series of very small puffs, crystals could be grown to any size. If it is fed too rapidly, crystals with re-entrant angles are produced. After the formation of the crystals, the CO_2 is shut off and the Dewar is raised to cool the plate still further and so prevent sublimation. Often a "snowstorm" of small CO_2 crystals occurs, covering and obscuring the larger crystals and causing a mottled appearance of the pictures. After the temperature had risen high enough to vaporize the CO_2, small heaps of ice remained which, on further warming, ultimately sublimed in the dry atmosphere or melted when large. None of such size, however, were produced in making the regular runs, as the quantity of gas used was small and the water accordingly minute. Considerable time elapsed between the evaporation of the CO_2 and the disappearance of the water, showing the difference in temperature.

If the entire chamber is too cold, and the CO_2 is turned on, a veritable CO_2 snowstorm occurs which is worthless for our purpose. If the plate alone is very cold and a great puff of gas is turned in, "spiderweb" forms may be produced.

Some clear patches were often observed containing no crystals. These were caused by a sort of explosion. Unfortunately, the eye was drawn to a point only after the explosion was over, so that what had really taken

To face page 70.

Fig. 3.

Fig. 4.

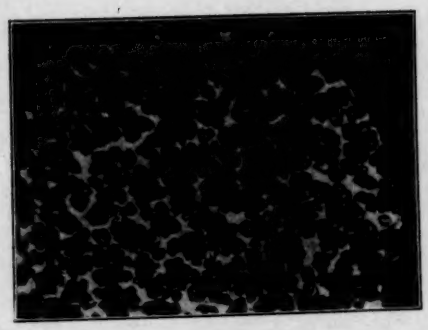

Fig. 5.

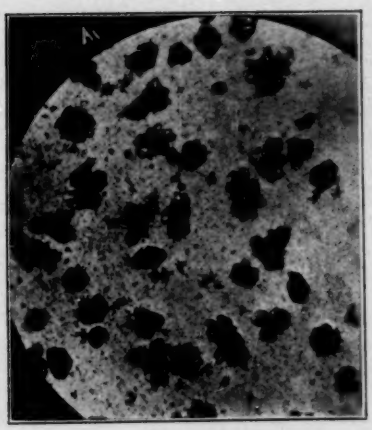

Fig. 6.

H. E. BEHNKEN.

PLATE I.

place could not be learned. It was probably caused by the cooling of a large CO_2 crystal causing its breaking due to contraction. It might be a change of crystal form taking place with a change of temperature. A comparison of three successive plates showed the result of these explosions. Clear spaces in locations where large crystals were shown on plates taken first were noted on the subsequent ones. The sharpest, best-formed crystals seemed most likely to rupture in this manner.

The CO_2 crystals appear to be combinations of cubes and octahedra. Many of the triangular faces with the corners off were clearly seen. Some square faces with the plain triangles are also observed.

On analysis, the gas purified in the way described showed an impurity of 0.16 per cent. unabsorbable in KOH. This percentage has been corrected for vapor tension of water, and calculated on the basis of zero water content of the gas analyzed. The unabsorbed residues of all the determinations were run together and the sum used in obtaining this result.

The work so far described was done in the summer session of 1909 and 1910. The following results were obtained in the summer of 1911.

It was deemed essential to use polarized light in the further study of the crystal forms. A polarizing nicol introduced into the freezing tube below the plate would, in all probability, have broken on cooling. Some compact device that would survive cooling, and if broken could be replaced readily, was needed. A pile of microscope cover glasses ground to fit replaced the concave mirror below the freezing plate. The reflecting prism was moved down and so set that the angle of incidence on the pile of plates was 57°; the plates being set at an angle of 33° in the tube so as to send the polarized beam directly upward. The rest of the lighting device was unchanged. The analyzer was a regular microscope nicol set above the ocular. A diaphragm made of black paper was used below the first reflector so as to prevent stray light and the inside of the Dewar was coated with dead black to prevent reflection. The freezing chamber and the end of the objective were blackened to avoid reflection of light upon the crystals. The small amount reflected from the object lens could however be noticed.

The darkening of field and CO_2 crystals was simultaneous, and no optical peculiarities could be noted. Some excellent crystals, perfectly clear and with perfect faces were produced.

Professor Gill was kind enough to visit the laboratory and to make a study of the CO_2 crystals. He declared them to be undoubtedly isometric and their forms to be combinations of cubes and octahedra.

Use of the Apparatus for Other Gases.

Ammonia (NH_3) and nitrous oxide (N_2O)[1] were used because their freezing points were near that of CO_2.

The N_2O was dried by passing it through P_2O_5, the same agent being used in the boat in the freezing chamber. The NH_3 was dried by passing it through a tube filled with metallic sodium threads formed by squeezing the metal through a die. The boat was filled with scraped sodium. For this method of drying, I am indebted to Dr. H. B. Browne.

The N_2O crystals formed readily when no moisture was present. When moisture was present or when the gas was fed too rapidly, lint-like forms were produced. On close examination, the threads looked somewhat like coral; on the whole, a formless bunch.

The leaf forms were very easily obtained, but the separate forms photographed were produced only with very slow feeding of the gas after the plate was extremely cold.

Ammonia crystals were very hard to obtain. If the plate were not cooled to a very low temperature the gas first liquified, and then froze into a solid mass. It was found necessary to submerge the freezing tube until the prism and the polarizing plates were almost completely submerged in the liquid air, whose level was not more than 2.5 cm. below the freezing plate. As a result, the polarization was poor and could not be used.

When the pure gas was fed into the cold chamber, a great mass of "ammonia snow" looking much like a mass of cotton was produced. It was very much like the "coral" formed by N_2O. Its method of growth was interesting. A cloud of tiny crystals would appear and fall on the plate. Others would fly into the field and alight on the first comers as though attracted by electric charge. This piling up in single column would go on with occasional branching, each fiber meanwhile swaying about wildly in the field of the microscope.

The crystals photographed were obtained by diluting the dry NH_3 with pure, dry air, obtained from the auxiliary system and feeding this diluted gas slowly into the cold chamber. In both NH_3 and N_2O isometric crystals were obtained. The photographs shown were selected from a considerable number to show the general appearance of the plates. The results obtained with these gases would indicate that with sufficient care in manipulation, the apparatus might be used for determining

[1] N_2O, melting point 102.3°, boiling point 89.8°; NH_3, melting point −75.5°, boiling point −38.5° (760 mm.). (Landolt-Bornstein's tables, third edition.)

the crystal form of many gases whose freezing points are only moderately low.

The writer is indebted to Professors Gill, Gage and Chamot for advice, and to Professor Shearer, under whose direction the work was carried out, he wishes to express his gratitude.

CORNELL UNIVERSITY,
August, 1911.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE EFFECT OF THE CHARACTER OF THE SOURCE UPON THE VELOCITIES OF EMISSION OF ELECTRONS LIBERATED BY ULTRA-VIOLET LIGHT.[1]

BY R. A. MILLIKAN.

SOME time ago[2] I presented to the Physical Society a brief description of experiments in which the positive potentials acquired by metals under the influence of ultra-violet light were greatly in excess of those obtained by other observers. The results thus far obtained in the further study of this "high-speed" emission may be summarized thus:

1. As in the case of the low-speed emission previously studied the positive potential acquired by the illuminated body appears to be completely independent of the intensity of the source. This result has been tested for intensities which vary in the ratio 15 to 1, but not as yet for intensities of wholly different orders of magnitude (*cf.* below).

2. As indicated in the original report the maximum positive potential acquired, as well as the photocurrent at any potential, is a function of the condition of the illuminated surface, and, in general, can be increased by prolonged illumination. The effects of continued illumination alone have been briefly reported in a recent paper.[3]

3. The velocity of emission is a function of wave-length, but apparently a different form of function from that found in the case of low speed emission.[4]

4. The velocity of emission is a function of some characteristic of the source other than wave-length and can be pushed up as high as to that corresponding to a positive potential of 500 volts or more by suitable changes in the character of the source of the ultra-violet light.

The present studies have to do only with 1 and 4. High speed emission is obtained only with *spark* sources, and variations in the electrical constants of the spark circuit (capacity, self induction, spark length, energy input) are found to produce large variations in the potential-photocurrent curves. The characteristic difference between the type of curve obtained with a quartz-mercury-lamp source and a spark source is shown in the accompanying figure,

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

[2] PHYS. REV., 30, p. 287, 1910.

[3] Millikan and Wright, PHYS. REV., Feb., 1912.

[4] J. R. Wright, PHYS. REV., August, 1911.

which is one of a large number of curves taken with unanalyzed light. By suitable variation of capacity and self-induction spark-source curves of the type shown in the figure can be made to pass over into curves of the mercury-source type.

In order to determine whether the high velocities obtained with spark sources might not be due to very short ultra-violet rays, or to other sorts of rays not given out by a quartz mercury lamp, the following tests were made. First, the absorption coefficients of the effective rays from both sources were compared and found to be essentially alike. This together with the fact that very thin films of either mica or benzole were found to cut off completely all effects from spark sources as well as from mercury lamp sources, indicates that the effective rays consist of ordinary ultra-violet light of wave-length between 260$\mu\mu$ and 180$\mu\mu$.

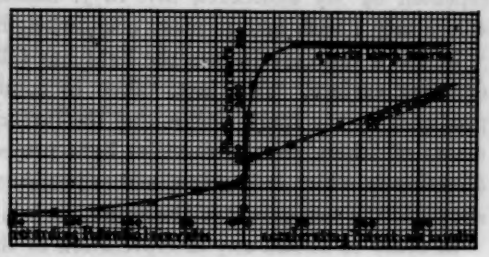

Fig. 1.

Second, monochromatic light of a given wave-length, obtained with the aid of a quartz spectroscope, shows the same characteristic difference between spark and mercury arc sources as that shown in the figure.

Third, spectroscopic studies of the results of changes in capacity and self-induction with spark sources showed no new ultra-violet lines and no changes whatever in the distribution of the light in the ultra-violet spectrum, while the positive potentials showed enormous changes.

In order to test whether all the differences between spark and mercury-arc sources might not be accounted for by assuming that, while ultra-violet light is necessary to release the electrons, the high velocities imparted to them by spark sources are due to imperfect screening from the intense electromagnetic disturbances of long period which go out from the spark, the following experiments were performed:

First, the whole spark producing apparatus was completely enclosed in a box made of sheet-iron, in one case 5 mm. thick and in another case 2 mm. thick; but this screening produced no essential change in the potential photo-current curves.

Second, the spark and mercury sources were placed so that the light from both could fall simultaneously upon the electrode from which the electrons were to be discharged, then a small screen was inserted so as to cut off the light from the spark source, which, however, was kept running while the mercury source curve was being taken. This was to enable the long electromagnetic waves from the spark to impart high velocities to the electrons liberated by the mercury source provided they were able to do so. *The velocities were in all cases those characteristic of the mercury source alone.* Just as soon however as the screen was removed so as to let the light from the spark, in addition to that

from the mercury arc, fall upon the electrode *the high velocities characteristic of the spark source were obtained.* This seems to show conclusively that the high velocities are due to the light itself, and not to secondary effects of any kind.

The above results have been duplicated for the most part with four different tubes constructed in different ways. Most of the observations too have been repeated by my assistants, Mr. L. J. Lassalle and Mr. A. F. Melcher, for whose help I wish here to make acknowledgment.

In all of the experiments the light has fallen normally upon the electron-emitting surface. This fact, taken in connection with the high velocities produced by the spark source, and the further fact established by the experiments of others, that ultra-violet light projects electrons most readily in the direction in which it is going, makes it easy to account for all the differences shown in the two curves of the figure.

In order to obtain any interpretation of the foregoing high velocities from spark sources it seems necessary either to discard conclusion 1 above, and assume that with such enormous differences in intensity as exist between the light from the mercury arc and that coming at certain instants from a spark source, the velocity of emission of electrons is not independent of intensity; or else to assume that light possesses some other quality besides wave-length and intensity which is responsible for these differences. The latter assumption would be extremely radical. The former is not yet justified by other experimental evidence. If it is permissible to make it, one of the strongest arguments for a "light-unit" hypothesis is removed. In any case these results are completely at variance with the Planck-Einstein light-unit theory in so far as the maximum value of the energy of discharge of electrons by ultra-violet light is concerned.

It should be pointed out that the striking similarity between the curves here obtained with the spark sources of ultra-violet light and the potential-discharge curves obtained by Seitz with the use of X-rays furnishes another indication of an intimate relationship between light and X-rays.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO.

NOTE ON THE VARIATION FROM LAMBERT'S COSINE LAW OF THE EMISSION FROM TUNGSTEN AND CARBON AT GLOWING TEMPERATURES.[1]

BY A. G. WORTHING.

IN the investigation of the conduction losses in incandescent lamps[2] an unexpected variation from Lambert's Cosine Law was noticed. A special investigation making use of the same method, though considerably refined for this purpose, has been undertaken.

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

[2] Hyde, Cady and Worthing, Trans. Ill. Eng. Soc., 6, p. 238, 1911; Ill. Eng. (London), 4, p. 389, 1911.

For a uniformly heated cylindrical tungsten filament viewed normally to its axis, the variation from the cosine law is such as to make the average brightness for $\lambda = .63\mu$ about 3.0 per cent. greater than that of the central portion; for $\lambda = .46\mu$ the variation is considerably less; for an untreated carbon filament, the variation is such as to make the average brightness about 5 per cent. less than that of the central portion. A change of temperature for the tungsten shows a definite change in the variation from the cosine law. The light emitted from the edge of the carbon filament was found to be about 20 per cent. polarized, that from the edge of the tungsten filament about 60 per cent. polarized, both in planes parallel to the axis of the filaments. These results for tungsten are similar to results obtained by Uljanin for Pt, Ag and Cu.[1]

Some Further Results of Precision Measurements of Refractive Indices as a Function of Temperature.[2]

By F. A. Molby.

THIS paper is a continuation of the work on crown glass, flint glass, and quartz, as reported at the Washington meeting, Dec. 29, 1911. An account of the measurements will be given in a future article in the Physical Review.

Rotations in the Metallic Arc.[3]

By W. G. Cady.

THE existence of rotations at the anode of the iron arc in air was pointed out in a former paper.[3] Further investigation has shown that the following phenomena occur when the arc has burned long enough to settle into a steady state, with a globule of molten iron oxide on the positive terminal. As cathode, iron or carbon have usually been employed, though any other metal may be used as long as it does not cause too great a change in the composition of the positive globule. To keep the globule sufficiently hot, it is best to use as anode an iron rod only a few millimeters in diameter, or else to place the globule on the end of a carbon rod.

At small values of current the positive globule, though liquid, is not in a state of vaporization, and the discharge at the anode is a glow. This constitutes what I have called the "first stage" of the arc.[4] At about 1.5 amp. the globule commences to vaporize and the current suddenly increases slightly. It is here that the rotations generally begin. If a greatly magnified image of the arc is thrown on a screen, a small bright ring is seen on the anode, indicating

[1] Ann. der Physik., N. F., 62, p. 528, 1897.

[2] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

[3] Cady and Arnold, Am. Jour. Sci., 24, p. 383, 1907.

[4] Cf. Hagenbach and Veillon, Phys. Zeitschr., 11, p. 833, 1910, and Fabry and Buisson, Jour. Phys. (4), 9, p. 929, 1910.

that the positive base of the discharge is rapidly describing a circular path. At the same time a very high-pitched whistle is heard.

The arc has been photographed on a rapidly moving film, which shows clearly the rotation of the positive base. Small fluctuations of current and p.d. have been simultaneously recorded with the aid of a high-frequency oscillograph. As the current is further increased the frequency of the rotations increases, reaching a maximum at about two amperes and then falling again, until at three or four amperes the rotations become too irregular to follow. The frequency is usually between 3,000 and 8,000 per second. The rotations can also be seen clearly with the aid of a rotating mirror. They cease in the absence of oxygen in the surrounding gas, and they occur at a smaller current if oxygen is present in excess. These facts, together with the further fact that the positive drop for iron oxide is greater than that for metallic iron, lead to the following explanation of the phenomenon.

If oxygen is present in the surrounding gas, the electronegative oxygen ions are driven to the positive base of the arc, causing the potential drop to increase. The positive base moves aside to a position where there is less oxygen, and thus, thanks to the rounded surface of the globule, it falls into a circular path. The diameter of the path depends on the velocity with which the base moves, and on a central force which prevents its wandering further from its mean position. This central force is furnished in part at least by the tendency of the arc to shorten.

In the equation for circular motion,

$$r = \frac{v^2}{a},$$

r, the radius of the path, and v, the velocity of the positive base, may be observed, and thus the value of the normal acceleration a may be determined, for various values of current and of arc length. It is hoped that the results obtained in this way may throw some light on the conditions prevailing at the anode.

A somewhat similar movement of the positive base is also present, though less clearly, in the case of anodes of nickel or cobalt. Other substances so far tried have failed to show the effect.

In the neighborhood of two amperes, the minute rotations just described often change abruptly to a much larger and slower rotation, of frequency between 700 and 2,000. The ring formed by the positive base is now about a millimeter in diameter, and can be seen directly through a red glass. This form of rotation is more difficult to obtain, and is stable only over a small range of current. When once obtained, however, it is very conspicuous, and it is this form of rotation which was referred to in the earlier paper. It is apparently due to some change—possibly impurities—on the surface of the globule, causing a change in the velocity of the positive base or in the acceleration. A more complete account of this effect will be published later.

Wesleyan University.

THE LINEAR EXPANSION OF INVAR STEEL, BETWEEN 100° C. AND −190° C.[1]

BY F. A. MOLBY.

THROUGH the kindness of Professor J. S. Shearer, a specimen of "Pendulum Steel" has come into the writer's possession, from which he has prepared a specimen 1.366 cm. in length, after the manner of Scheel, Dorsey and others, and, by the method of interference of monochromatic light in vacuum space, the writer has studied the linear expansion of this ring specimen between the temperatures of steam and of liquid air. The maker's description or guarantee of this sample of steel is not known to the writer, but the properties which it is found to have must place it among the high class invar steels. Mr. E. C. Guillaume[2] gives data for one piece of nickel steel, 36.1 per cent. nickel, which has the equation

$$L_t = L_0(1 + 87.7 \times 10^{-8}t + 12.7 \times 10^{-10}t^2) \qquad (1)$$

between 0° C. and + 38° C. In another place[3] he describes a piece of steel that has a coefficient of expansion so small as not to be measurable with any degree of certainty. The specimen which the writer has used is found to have the following equation between length and temperature, for the range from 100° to −190°:

$$L_t = L_0(1 + 102 \times 10^{-8}t - 22.3 \times 10^{-10}t^2 + 1 \times 10^{-12}t^3) \qquad (2)$$

which differs comparatively little from the values given in the equation (1) cited above, but it does differ in that the algebraic sign of the coefficient of the t^2 term is negative instead of positive.

Two sets of measurements were made, giving agreement for the change in length within about two per cent. for the entire range. In the accompanying table are given the mean of the values observed, as scaled from a curve in which all observations were plotted, reckoning from a temperature of zero degrees.

Temp. Range.	Fringes Shift.	δL	$\delta L/L_0$
0° to 20°.0	0.95	0.0000259	0.0000190
0 to 100.	4.08	0.0001090	0.0000799
0 to −100.	− 6.26	0.0001709	0.0001250
0 to −190.	−14.06	0.0003840	0.0002811

The fixed temperatures as steam, room, zero, and liquid air were held constant for such a length of time, one to one and one half hours, that the specimen had entirely come to a steady temperature. The pressures of the air in the optical path through the ring were very nearly one half millimeter, and certainly not so much as one millimeter. The wave-length of light used is 0.00005461 cm., and a shift of one fringe therefore corresponds to a shortening, or lengthening, as the case may be, of the specimen by the amount 0.0000273 cm. That the

[1] Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

[2] Comptes Rendus, Vol. 124, p. 178, 1897.

[3] Comptes Rendus, Vol. 136, p. 303, 1903.

specimen actually contracts on cooling is known because the fringes shift toward the same edge that they shift toward when the air is being exhausted from the tube in which the specimen is placed. The column entitled δL in the above table gives the change in length for the 1.366 cm. of steel for the corresponding temperature range: the column entitled $\delta L/L_0$ gives the change in length per cm. length for the corresponding temperature range.

In the accompanying curve it may be seen that the length L_t of the specimen is not a linear function of the temperature, but it is quite accurately expressed by the equation (2) given above. A second degree equation might have been determined which would be sufficiently accurate. In the equations of Mr. Guillaume, the coefficient of the t^2 term is positive in all cases noted by the present writer, which would lead us to expect that the effect of temperature should be relatively less at low temperatures, as in fact it is in the case of the greater number of materials whose low temperature coefficients of expansion have been determined; by the usual way of defining coefficients of expansion as

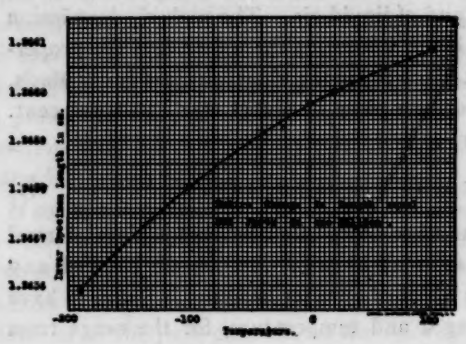

Fig. 1.

$$\alpha_t = \frac{\frac{dL}{dt}}{L_0},$$

several values of α may be found by differentiation of equation (2) and by substituting the temperature for which the value of α is desired. In the following table are several such values, dL/dt being the slope of the length temperature curve at the temperature for which the value of α is given.

Invar.		Platinum.	
Temp.	α_t	Temp.	α_t
100	60.4×10^{-6}	100	9.585×10^{-6}
0	102.0×10^{-6}	0	8.749×10^{-6}
−100	149.6×10^{-6}	−100	8.329×10^{-6}
−190	197.6×10^{-6}	−190	8.308×10^{-6}

By comparison with the values derived from Scheel's equation for platinum, it is seen that the invar specimen, although expanding by many times smaller amounts, is not so regular in its expansion as platinum has been found to be.

CORNELL UNIVERSITY,
April, 1912.

[1] Karl Scheel, Deutsch. Phys. Gesell. Verh., 9, pp. 3–23.